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Emerging Bilinguals' Mathematical Agency in a Teaching Experiment: Tomar posesión y entender las ideas matemáticas

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Emerging Bilinguals' Mathematical Agency in a Teaching Experiment: Tomar posesión y entender las ideas matemáticas

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Dedication

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Abstract

Emerging Bilinguals' Mathematical Agency in a Teaching Experiment: Tomar posesión y entender las ideas matemáticas

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Latino/a emerging bilinguals with identified learning disability labels face marginalization in the mathematics classroom due to several barriers, such as the type of instruction and denial of native language as a resource. Little research is devoted to helping Latino/a emerging bilinguals with identified labels have access to mathematical practices that promote mathematical understanding, and therefore small opportunities to enact mathematical agency. This dissertation offers an alternative to current mathematics instruction by examining the extent at which three Latino/a emerging bilinguals identified with struggling or learning disability labels in elementary school exhibited mathematical agency through participation in mathematical discussions centered on problem solving. The three Latino/a emerging bilingual children were engaged in discussions of base ten and fraction concepts coupled with mathematical practices in a teaching experiment. The findings indicate that these children have the capacity to enact mathematical agency when exposed to problem solving discussions that position them as competent learners. The

findings also indicate children are were more likely to participate in meaningful discussions when allowed to use their mathematical thinking and ideas.

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Chapter 1: Introduction

Research indicates that participating in mathematical practices supports children's conceptual development and bolsters achievement in the classroom (Bodovski & Farkas, 2007; Webb, Franke, Ing, Wong, Fernandez, Shin, & Turrou, 2014). This research defines mathematical practices as problem solving, having discussions with others about problem-solving strategies, and justifying solution strategies. Educational policy documents in the United States align with and support the importance of a focus on mathematical practices alongside mathematics content. In fact, the Common Core Standards for Mathematics (CCSS, 2010) recommends that all children in K-12 mathematics classrooms engage in "making sense of problems", "constructing viable arguments", and "critiquing the reasoning of others" (CCSS, 2010, pp. 6).

Recommendations from research and policy for classroom instruction are intended to include all children. Yet, far too often, Latino/a emerging bilingual children, defined as Spanish speakers who are learning an additional language, usually English (Garcia, 2009; Garcia & Kleifgen, 2010), are denied access to mathematical practices due to barriers they face in instruction. For instance, Latino/a emerging bilingual children are often asked to speak only in English, thus disadvantaging these children by not allowing them to use their primary language as a resource during classroom discussions (Cummins, 1986; Planas & Civil, 2013; Khisty, 1995). Moreover, Latino/a emerging bilingual children are asked to work with tasks or contexts that are unfamiliar to them or detached from their home or community environments (Trueba, 1998). The tasks become irrelevant, making it difficult or even impossible for these children to make connections between the mathematical ideas in the task and the knowledge they do in fact possess. Additionally, they are often removed from social environments that could help construct their mathematical understanding and agency and placed in reductive instructional settings (Lampert, 2015). Finally, Latino/a emerging bilinguals often work in instructional environments that are limited in the availability of resources to help in enacting mathematical practices (e.g. teachers implementing reform oriented mathematical pedagogies) (Skiba, 2013).

Latino/a emerging bilingual children who experience the culture and systems of American schools in these ways over time oftentimes "act" in ways to assimilate. Valenzuela (1999) describes a type of resistance wherein children's behavioral patterns become mentally and physically absent, a type of refusal to participate in mathematics class activities. The school culture, social interactions, and economic conditions limit choices children have to connect mathematics to their worlds. Cavell (2011) attributes Latino/a children's agency as a form of resilience to a feeling of apprehension and misunderstanding when mathematics instruction is "devoid of connections to their real social worlds" (p. 63). Children usually do what they are told to do, and do not challenge the status quo.

Barriers to accessing mathematical practices in the classroom are often reflected in the mathematics performance of Latino/a emerging bilingual children. National statistics reflect that Latino/a emerging bilingual children present achievement gaps in mathematics early in their school experience (National Center for Education Statistics, 2016). Particularly, 4^{th}-grade mathematics National Assessment of Educational Progress (NAEP) scores show an achievement gap of 28 percentage points between non-English Language Learners (ELLs) and ELLs (of which most are Latino/a emerging bilingual children) (NCES, 2016). Unfortunately, if the performance gaps are sustained over time, the solution often proposed and implemented in schools is to identify these children as having a learning disability (LD) and place them into special education (Sullivan, 2011; Artiles, Trent, & Palmer, 2004; Artiles, Rueda, Salazar, & Higareda, 2005).

Latino/a emergent bilingual children need connections to their culture, home experiences, and native language to be able to succeed in the math classroom. In fact, many researchers recognize that there is a strong association between math achievement and Latino/a emerging bilingual children's native language, prior knowledge, and cultural experiences (Abedi & Gandara, 2006; Abedi & Lord, 2001; Borjian, 2008). Thus, these children will learn best when given opportunities to use their prior knowledge, language and cultural experiences in the mathematics classroom. Labeling Latino/a emerging bilingual children as disabled and placing them into special education programs does not remove barriers to accessing mathematical practices or increase low performance in meaningful ways. In fact, I argue the placement exacerbates the problem, works to increase barriers, and further marginalizes these children.

When Latino/a emerging bilingual children are set along the pathway of identification to LD, the prevailing assumption is the children need a particular type of mathematics to "remediate", or "fix", low performance (Freeman & Crawford, 2008; Orosco, 2014) before assigning a label of LD. The dominant model of remediation entails an explicit delivering of mathematics by teachers onto children (e.g. Garcia & Tyler, 2010), positioning mathematical knowledge as children's response to teacher-led instruction (Gersten, Chard, Jayanthi, Baker, Morphy, & Flojo, 2008, 2009). The focus is on children's memorizing and recitation of the teacher's modeled strategies and procedures (e.g. Brosvic, Dihoff, Epstein, & Cook, 2006; Freeman & Crawford 2008; Freeman 2012; Orosco, 2014). But this type of instruction does not address the problem because it further removes them from instruction that focuses on mathematical practices, such as sharing mathematical thinking strategies used to solve problems (Carpenter, Ansell, Franke, Fennema, Weisbeck, 1993; Empson, 2003; Turner & Celedon-Pattichis, 2011; Webb et al, 2014), and enact agency.

Furthermore, placing Latino/a emerging bilingual children into special education instruction focused on computation, fact retrieval, and memorization has not improved mathematics performance long term or provided increased access to a college degree (Gottfried, Bozick, Rose, & Moore, 2014; Shifrer, Callahan, & Muller, 2013; Wells, Sandefur, & Hogan, 2003). Shifrer, Callahan, and Muller (2013) determined that children with LD complete fewer academic courses by the end of high school when compared with children without LD. The researchers identified a significant proportional gap of 27% to 50% in all high school graduation courses when comparing children to LD with children without LD, reflecting that only 27% of high school courses that students with LD complete are academic in nature. Similarly, students with LD are less likely to attend college, leaving opportunities to pursue STEM-related fields less likely (Wells, Sandefur, & Hogan, 2003). For these reasons, I argue that this study will demonstrate a different path for Latino/a emerging bilingual children labeled LD with more possibilities to succeed during their K-12 math education years and potentially provide access to STEM careers.

An alternate approach to reductionist, teacher-led instruction and assumptions that low performance equates to a need for special education services is to *ensure access* in instruction that fosters participation in mathematical practices to build *mathematical agency* (Empson, 2003; Turner, Dominguez, Maldonado, & Empson, 2013; Yamakawa, Forman, & Ansell, 2009; Web et al., 2007, 2008). As intended by US policy (CCSS, 2010), all children, including Latino/a emerging bilingual children who present mathematical difficulties in the elementary grades, would benefit from engaging in mathematical practices (Yackel, Cobb, & Wood, 1991; Yackel et al, 1990). Furthermore, engaging

Latino/a emerging bilingual children in mathematical practices may also bolster their identities as mathematical thinkers and doers and contributors of mathematical ideas and arguments, thus helping them develop mathematical agency. Positioning Latino/a emerging bilingual children as agents of their own mathematical ideas could promote sustained access to advanced mathematics coursework in middle and high school (Shifrer, Callahan, & Muller, 2013), furthering these children's educational opportunities beyond K-12 settings.

Some researchers have adopted mathematical practices that focus on children's mathematical thinking and understanding (Moschkovich, 1999; Turner et al, 2013; Hunt & Empson, 2014). Mathematical practices have been used with individual children labeled LD (e.g. Empson & Hunt, 2014), with whole group classes of Latino/a emerging bilingual children (e.g. Moschkovich, 1999) and even in whole class discussions of Latino/a emerging bilingual children with math difficulties (e.g. Turner et al., 2013). For example, Moschkovich (1999) utilizes mathematical practices such as re-voicing, clarifying strategies, and gestures to help support emerging bilingual children in mathematical discussions. In her work, Moschkovich highlights the importance of emerging bilingual children participating in mathematical practices promoting understanding. Although this work provides an essential documentation of bilingual discourse on the mathematical practice of problem solving, the Latino/a emerging bilingual children were not children who had been continually marginalized and siloed with labels such as "at risk" or "learning disabled."

Turner and her colleagues (2013) used mathematical practices to position 34 fourth and fifth grade children who struggle learning mathematics as developing agentic roles (e.g. as problem solvers and mathematical justifiers of ideas) in a whole class teaching experiment; 22 of these children were ELLs. This study approximates the investigation of mathematical agency enacted by Latino/a emerging bilingual children who have been identified with struggling labels, but again does not capture this subset of children who are both emergent bilinguals and labeled with a learning disability. Hunt and Empson (2015) conducted individual clinical interviews with 10 third through fifth grade children with LD labels to uncover initial conceptions of fractions in the context of equal sharing word problems. While important, this work was done with children on an individual basis for the purposes of documenting the children's conceptual knowledge and was not a documentation of how these children might build mathematical agency.

What has yet to be explored in research is *how Latino/a emerging bilingual children already set along the trajectory of being labeled as struggling in mathematics or as having a LD (a) access to and participation in the mathematical practices and (b) regain and empower themselves as mathematical thinkers and doers through agentive participation*. I will document pathways by which engaging Latino/a emerging bilingual children identified as struggling in mathematics or LD can access mathematical practices and how these practices might support children in developing and enacting *mathematical agency* during a series of problem-solving discussion sessions. This research is important because it will address a gap in the literature about how Latino/a emerging bilingual children with identified math difficulties and LD labels enact and develop mathematical agency during mathematical discussions about problem solving situations. The mathematical content will center on previous research-based strategies that support all children's conceptual understanding of number and operation concepts (e.g. Fennema, Carpenter, Franke, Levi, Jacobs & Empson, 1998). I will explore the following research question:

How do Latino/a emerging bilingual children labeled with learning disabilities and/or difficulties develop mathematical agency as mathematical learners through the participation in mathematical practices?

Chapter 2: Literature Review

The focus of this study is on the *mathematical agency* that Latino/a emerging bilinguals with identified learning disabilities and difficulties enact as they participate in mathematical practices and how *mathematical* agency develops through the engagement of mathematical discussions of problem solving situations. The aim of this review is to draw on research and empirical work that makes evident how Latino/a emerging bilingual children become marginalized in the current instructional system, and in doing so, outline a conceptual framework that focuses on promoting *mathematical agency* for this population.

In the first section, I provide research that outlines the link between participation in mathematical practices and *mathematical agency*. I begin by examining the importance of mathematical practices for the sake of mathematical understanding. Then I discuss the construct of *mathematical agency* to delineate the framework used in this study to explain how agency is situated in these mathematical practices. Finally, I provide evidence that Latino/a emerging bilingual children are denied access to these mathematical practices and mathematical agency and discuss the pathway children often travel in schools as a result (i.e., identification as "struggling" and/or special education). In the second section, I construct an argument for how placement in special education exacerbates the problem, works to increase barriers, and further marginalizes these children thus removing the mathematical agency they could possess. In the third section, I review prior research on Latino/a emerging bilinguals with identified learning disabilities and difficulties in terms of what has been done to promote access to participation in mathematical practices and mathematical agency in mathematics classrooms and areas yet to be explored. Finally, I present linkages between the research questions in the current study and areas yet to be explored in research.

2.1 Mathematical Practices AND Performance

Over the past several decades, research has demonstrated the potential of mathematical practices to enhance children's learning and achievement (Chinn, O'Donnell, & Jinks, 2000; Fuchs, Fuchs, Hamlett, Phillips, Karns, & Dutka, 1997; Howe, Tolmie, Thurston, Topping, Christie, Livingston, et al., 2007; Veenman, Denessen, van den Akker, van der Rijt, 2005; Webb, Franke, Ing, Chan, Freund, Shein, et al; 2008; 2009; Webb et al, 2014). Mathematical practices include (a) engaging children in problem solving situations, (b) engaging children in discussing their mathematical thinking with others and (c) engaging children in justifying and elaborating their mathematical thinking (Webb et al., 2008, 2009; Howe et al., 2007). Research on children engaged in problem solving situations has documented positive shifts in children's conceptual understanding (Carpenter, Franke, Jacobs, Fennema, & Empson, 1998; Fennema et al., 1996). For example, Carpenter and colleagues documented how 82 children in the early grades (1-3) in a 3-year longitudinal study demonstrated use of invented strategies to solve word problems involving addition, subtraction and multi-digit operations. The children who used invented strategies (e.g. 13+37 is 50 because I know that 10+30 is 40 and 3 and 7 is 10 so I have 50) early in the study had "significantly fewer systematic errors than those children who began using algorithms" (Carpenter et al., 1998, p. 46-47). Some researchers (Howe et al., 2007; Veenman et al., 2005) have found that the simple act of children sharing their explanations promotes learning and increased mathematical outcomes to some degree. Chinn et al (2000) argued that it is not simply explaining but also the quality of explanations given by a child during group discussions that is highly predictive of outcomes of learning and mathematics performance. Other researchers (e.g. Chi, Bassok, Lewis, Reimann, & Glasser, 1989; Fuchs et al., 1997; Roscoe & Chi, 2008; Webb et al., 2014) assert that the act of children sharing complex explanations, the degree of elaborations, and the quality of justifications are strongly related to higher achievement. For instance, Webb and colleagues (2014) argue that far greater achievement gains are evident when children are engaged in justifying their ideas, challenging their peers, clarifying their ideas, and building on other's ideas, more so than when children simply explain their ideas to others.

2.2 Tying Mathematical Agency to Participation in Mathematical Practices: A Framework

How can participating in mathematical discussions centered around mathematical practices be a tool to give marginalized groups of children opportunities to enact mathematical agency? This is the essential question this study aims to investigate. The traditional view of doing mathematics enacted in schools can have limiting and marginalizing effects on Latino/a emerging bilingual children. Mathematics should be viewed as giving people choice to do and learn how to solve problems in ways that make sense to the individual, thus giving power of choice on forms of thinking and knowing, following recommendations from research. I argue that Latino/a emerging bilingual children with identified math difficulties' participation in mathematical practices during discussions of problem solving situations can foster what I refer to as *mathematical agency*.

In this section, I will discuss how *mathematical agency* is constructed. I begin by describing how previous work on agency defines agency with Latino/a children. I draw from the work of Pruyn (1999), Turner (2003), Pickering (1995) and Adair (2014) and outline the similarities and differences between the definitions of agency offered previously and the working definition I outline as *mathematical agency* for this study. Then, I will discuss how the construct of *mathematical agency* is linked to the participation in mathematical practices described.

Previous work on Agency with Latinas/os. Research in instruction that is focused on issues of equity usually looks for ways in which children can enact agency. *Agency* as defined by many is the capacity for children to make their own choices and to enact choices on the world (Holland, Skinner, Lachiotte, & Cain, 1998; Pruyn, 1999). Research in mathematics education on agency with Latinos/as is of particular interest to many scholars (Adair, 2014; Gutstein, 2003; Pickering, 1995; Pruyn, 1999; Sanchez-Suzuki Colegrove & Adair, 2014; Solórzano & Delgado 2002; Turner et al., 2013). Many of these researchers (e.g. Sanchez-Suzuki Colegrove & Adair, 2014; Pruyn, 1999) aim to defy deficit thinking by showing the possibilities of minority Latino/a children when given the opportunity to "act" on the choices they have to do mathematics.

Pickering (1995) refers to agency in the context of schooling, and categorizes it in terms of what children do in the classroom. In Gresalfi, Martin, Hand and Greeno's (2009) article on constructing competence, they describe Pickering's definition of *agency* as "engaged in decision making, exploration, and strategizing" (p. 53). In the context of the mathematics classroom, this *agency* could be interpreted as children creating and making sense of the mathematics.

Current mathematical experiences in schools are devoid of choices, thus greatly impacting children who are assimilating into American culture, particularly minority children. Thus, giving all children in instruction the power of choice becomes important. Adair (2014) refers to *agency* in her study as "in the context of schooling as the ability to influence and make decisions about what and how something is learned in order to expand capabilities" (Adair, 2014, p.217). Adair defines agency broadly to encompass the ability of young children to make their own choices in the early grades (e.g. Kindergarten) by allowing them to choose things such as the content taught, types of exploration and discourses, design projects, and use resources (e.g. texts, materials). This type of agency allows all children to explore multiple ways of learning beyond the traditional learning environment. Adair gives young children opportunities to expand their capabilities by offering them many resources in inquiry classrooms. She further investigates young children's power of choice to expand capabilities with Latina/o immigrants (e.g. Sanchez-Suzuki & Adair, 2014), to demonstrate the possibilities young Latina/o children have of thriving in learning environments when given a chance to enact an agency of choice.

Although the power of choice to expand capabilities is important, I argue there is a more pressing type of agency needed for marginalized children such as Latino/a emerging bilinguals. Thus, I argue Pruyn's (1999) definition of *critical student agency* is more appropriate. He defines *critical student agency* as "the purposeful action taken by a student, or group of students, to facilitate the creation of counter-hegemonic pedagogical practices" (p.21). Pruyn's definition is more appropriate because it directly places the action or agency onto the child, as opposed to the teacher providing a choice: the child now has the capabilities to enact it on their own despite their previous or current experiences in the classroom. *Critical student agency* allows Latino/a emerging bilinguals to be more conscious of their learning environment and gives them opportunities to create change in the way they learn. It allows them to change the way learning occurs in their classrooms.

While *critical student agency* would be a powerful tool for change among Latino/a emerging bilingual children, it is imperative that I point out some of its limitations. For example, Turner (2003) argued this type of agency is "not stable" but is instead "fluid…

of one's identity that develops and thickens over time" (p. 29). Such fluidity can be challenging in the mathematics classroom because Latino/a emerging bilinguals are constantly moved from their social learning environments to participate in more isolated math environments. This makes it difficult for Latino/a children to enact power of choice and create change in these kinds of mathematical instructional environments.

Mathematical Agency. I am interested in how we as math educators could give children opportunities to act in ways that promote "expanding capabilities" to learn mathematics, be agents in their own mathematical learning, not acting as passive receptors of knowledge but creators of it. We could provide Latino/a emerging bilingual children who struggle in math and/or have been identified with an LD label the ability to execute ideas, be creative, and take risks while engaged in doing math. This then disrupts what Boaler and Greeno (2000) identify as "traditional pedagogies and procedural views of mathematics combine to produce environments in which most (children) must surrender agency and thought in order to follow predetermined routines" (p. 171). I borrow from Turner's (2003) definition of *critical mathematical agency*, which she defines as children's

> "capacity to (a) understand mathematics, (b) identify themselves as powerful mathematical thinkers, and (c) construct and use mathematics in personally and socially meaningful ways" (p. 48)

Turner's definition of *critical mathematical agency* is critical for Latino/a bilingual children who have been continually marginalized and oppressed by the discipline of hegemonic pedagogical practices (Pruyn, 1999). While I agree with Turner's definition of agency and believe it not only critical to "understand mathematics" and "use mathematics" in meaningful ways, I also believe it is important for children to view doing mathematics as "sense making" and as socially constructed, thus leading children to use their prior mathematical thinking to produce new mathematical knowledge (Lave & Wegner, 1991). I believe Turner's definition is important, but for the purpose of this study I attend only to children's ways of understanding and use of prior knowledge to solve story problems in typical contexts. I attend to Turner's critical math agency by utilizing the first two parts of her definition, the "understanding" of the mathematics presented and the potential for children to see themselves as "powerful mathematical thinkers".

Why is mathematical agency important? It is important because it provides traditionally marginalized groups of children opportunities to thrive in enacting or developing mathematical agency in pedagogies centered on mathematical practices that promote learning. Thus, I define *mathematical agency* as children's *power* to (a) make sense of mathematics and (b) take ownership of their mathematical thinking, where making sense of the mathematics is directly linked to children being able to understand their mathematical solutions with the given math problem's context (e.g. story word problems). In making sense, children are able to make a connection between the solutions they used and the math problem. In this definition, children's power to take ownership of their math thinking is directly linked to a child taking an action on their mathematical thinking. Boaler & Greeno (2000) further explain that a component of agency is self-motivation, which is intrinsic in nature but nonetheless important in any learning environment. It is especially crucial for Latino/a children who have been identified with math difficulties to take ownership of their mathematical thinking and as such there needs to be a deep-rooted choice to "act" on their mathematical ideas. I would like to point out that enactment of mathematical agency is not individualistic but rather socially negotiated and constructed in an environment where children are given opportunities of choice (e.g. the solution they would like to execute, how they would like to communicate ideas) with specific teacher moves that promote children's competence as mathematical experts and evaluators of ideas.

Children could therefore enact *mathematical agency* when they begin to take a greater role in their mathematical sense making. For example, in enacting mathematical agency children use innate ways of knowing to solve mathematical problems that entail their thinking. Suppose a child is working on base ten problems, and is asked to solve a multiplication word problem with five groups of 12 candy bars and the child chooses to use a buggy multiplication algorithm (e.g. 12×5, first I have to multiply the 2 and 5 so that gives me 10, ok, so now I place the 0 on the bottom and then 1 on top of the 12 next to the 1 so then I multiply 5 times 1 and multiply or add the 1 on top so that gives me 50, yes I think it's 50) to solve the problem. In this example, the child is more worried about performing the algorithm appropriately than about the context of the problem and what the numerals represent. In a sense, a child in this example is not making sense of the mathematics but instead focused on an algorithm that he/she believes is more important; thus the degree of agency is limited. Learning to make mathematical conjectures and relationships is far more important than learning a buggy algorithm in the real world and as such is something we should try to instill in all children.

Now, suppose the child chooses to draw on his/her own informal knowledge to create a strategy that makes sense to the child. Instead of using a buggy algorithm, a child chooses to use circles to represent the five groups and dots to represent the 12 candy bars. Here the child is no longer worried about procedures but instead is trying to make sense of the context and use what he/she does know, which is counting by ones, and making groups of ones. This affords the child possibilities to make sense of the mathematical concepts and connect other units (groups of 12 ones is a unit of 12 and I can count by 12 with enough experience). This allows this child to later understand how to build composites with enough experience (i.e. I understand that 12 is a unit and it is made up of 12 units of one). When compared to the buggy algorithm, the robust strategy of making circles and dots in the long run allows children to make meaning of the mathematical concepts. Thus, the robust strategy allows the child to have power to make sense of the mathematics and understand the underlying concepts. Thus, a higher degree of mathematical agency could be attributed to this child.

Children also may begin to try to make sense (understand) their solutions and explain their thinking to others. Furthermore, children may begin collaborating with others to solve and understand the mathematical problems at hand. For example, a child may be struggling to solve the above multiplication problem of five groups of 12 candy bars and asks a peer how they could potentially work together to solve the problem. Both children contribute mathematical ideas and brainstorm to try to make sense of the word problem. Mathematical ideas are co-constructed among children where the goal is to create a solution that addresses their interpreted understanding of the given word problem, thus both children exhibit higher levels of mathematical agency.

In addition to making sense of the mathematics as an important element in building *mathematical agency* for a child, so is taking ownership of their mathematical thinking and ideas. When children begin to take ownership of their mathematical thinking, they become confident in taking actions on their mathematical thinking. Take for example, a child who is used to following rules and procedures in the classroom, he/she is always expecting the teacher to take control of his/her mathematical thinking, allowing the teacher to provide examples similar to his/her math problems. Further, the teacher is eliminating the opportunity for the child to have a productive struggle in having to come up with a strategy to solve a mathematical problem. In this example, the child has very few capabilities of taking ownership of his/her strategies because they have been contrived by the teacher. Therefore, the child has a limited possibility to build his/her own strategies and therefore no opportunity to take ownership of his/her mathematical ideas and thus exhibit limited *mathematical agency*. In contrast, to the previous example, what if the child does have opportunities to create his/her own strategies and share and discuss these strategies with others? A child could develop a sense of ownership over his/her mathematical ideas. For instance, the child could potentially be open to taking action by sharing silently with gestures or verbally justifying, and arguing their mathematical ideas. This child will be more open to questions from teacher and peers than when their strategy does not make sense to them. The child will have opportunities to act on his/her thinking and explain why he/she thought the strategy he/she created worked and made sense to him/her. In addition to being able to share and justify their mathematical strategies and ideas, the child might begin to question what their peers create as their mathematical strategies and explanations. For instance, a child may be curious to see how his/her peers solved the same word problem (e.g. 6 candy bars shared with 5 people) where this child obtained a solution by splitting each candy bar into halves and then splitting the two leftover halves into five each. A peer instead solved the problem by sharing one whole candy bar per person and splitting the leftover whole into five parts. This child's curiosity may lead to asking questions about why their shares looks very different and if every person got the same amount of candy bar despite them splitting the candy bars very differently. Here the child views their mathematical solution as valid and tries to interpret why his/her peer's solution is different than his/her solution by asking questions to their peer. This example illustrates a child having ownership by trying to take an action on their mathematical thinking in the form of questioning and then interpreting the different solutions.

I would like to clarify that both the power to make sense of the mathematics and taking ownership of their mathematical thinking are both essential in building mathematical agency. You cannot have one without the other. Let's suppose that a child above notices that a peer's strategy is very different from his/hers. Now if this does not prompt the child to take any action on this, taking his/her mathematical ideas as making sense and not questioning or inquiring about their peer's solution, this would be viewed as *developing mathematical agency*. Thus, in group discussion a child may say that he or she agrees with a peer's ideas but does not try to question or understand their peer's reasoning. Now let's suppose that a child refuses to engage with the ideas of their peers by refusing to take any action (e.g. interpreting or trying to understand a solution) on a peer's mathematical thinking, this child's refusal would be seen as *limited mathematical agency*. This child has a degree of agency is in the form of refusing to engage with other's ideas.

In contrast, take for example a child that does not come up with their own strategy (borrows it from a peer) and it does not necessarily make sense to them, but decided to share with others. Does this child exhibit some form of mathematical agency? I would argue this could be considered *limited mathematical agency* because this child is not taking an action on his/her mathematical thinking, that is, this child simply takes someone's ideas without trying to interpret or understand these ideas, thus exhibiting little sense making. Now let's suppose the child used a standard algorithm but was able to make sense of the mathematics by asking peers to explain and then taking an action by interpreting and discussing with peers their interpretations of the algorithm, then in this case, yes, the child would be exhibiting mathematical agency because he/she would have the power to make sense of the mathematical concepts at hand.

Mathematical Agency in Mathematical Practices. Mathematical agency could be generated by the mathematical practices that promote learning. Mathematical agency could be created through problem solving and mathematical discussions where children are given a choice to make sense of problems, construct viable arguments, and critique the reasoning of others. Carpenter et al., (1998) explain how children who use invented strategies to "make sense" of the mathematics in very intrinsic ways are more successful in achieving deeper understandings of the mathematics than the children who enact standard algorithms. In this example, children are given a choice to enact a strategy that makes sense to them, and this choice is one of the components of *mathematical agency*. In this example, mathematics is no longer viewed as abstract and devoid of Latino/a children's culture and prior knowledge, and I argue that now children view mathematics as a pathway to make sense of the world around them. Mathematics becomes a tool which Latino/a emerging bilinguals can use to empower their thoughts and experiences. Similarly, in research (e.g. Web et al., 2014) focused on the discourse that children were able to use to make sense of the mathematics in more meaningful ways. These researchers found that constructing viable complex arguments produces higher levels of mathematical knowledge. In this example, I argue that constructing viable complex arguments goes hand in hand with developing mathematical agency because children are given opportunities to speak up and share their complex thoughts, thus producing a sense of belonging in this community. Children's ideas matter in this learning environment and thus increasing and developing their mathematical agency. Not only are solving math problems in ways that make sense and discussing their mathematical ideas with others important in helping

Latino/a Emerging children with identified math difficulties develop agency, but so are social learning environments where children can critique the mathematical reasoning of others. It is this change in mathematics pedagogy that produces "acts of power", that creates this social construction of mathematical agency.

2.3 Access to Mathematical Practices for Latino/a Emerging Bilinguals

Mathematical practices that promote understanding are supported by research and policy and are intended to include all children in US schools. However, due to school-level barriers (i.e. type and quality of instruction due to social factors, such as teaching to tests; denial of native language as a resource; unfamiliarity of the context; exclusion of home experiences; and effective use of resources), Latino/a emerging bilingual children are often denied access to mathematical practices in classrooms. I unpack each of these barriers in the paragraphs that follow.

Type and quality of instruction due to social factors. It is the assumption that all children are receiving the same type of instruction noted in the Principals and Standards of Mathematics, which states that *all* children "regardless of their personal characteristics, background, or physical challenges, can learn mathematics when they have access to high-quality mathematics instruction" (Executive Summary, The National Council of Teachers of Mathematics, 2000, p.2). In addition, the process standards for problem solving states that *all* children should "be encouraged to reflect on their thinking during the problem-solving process so that they can apply and adapt the strategies they developed to other problems" (NCTM, 2000, p.4).

What is troubling is that Latino/a emerging bilingual children are part of a "growing inequality" in terms of "access to science and mathematics education" (National

Science Board, 2006). Many of these children come from low socio-economic backgrounds (SES); unfortunately, these children are often engaged in instructional tasks that consist of doing worksheets or following step-by-step examples from a mathematics textbook, with no opportunities to reflect on their thinking (Swanson & Stevenson, 2002). Camburn and Han (2011) found that children from low SES backgrounds had less instructional time and coverage of mathematics textbooks and workbooks along with fewer conceptual and problem solving instructional experiences. These kinds of experiences are far removed from the kinds of instruction called for in research, policy, and curriculum standards that work to promote understanding and *mathematical agency*.

Several other researchers found that classroom teachers of Black and Latino children from low SES backgrounds spent most of their instructional time assessing mathematical 'success' (e.g. test grades, Lubienski, 2008) by completing minimalistic tasks, such as correcting right or wrong answers, as a form of accountability in the classroom and less time on high level tasks (e.g., mathematical argumentation; participation in inquiry tasks) (Anyon, 1981; Garcia & Gonzales, 2006; Gandara & Contreras, 2009; Genishi & Dyson, 2009; Heilig, 2011; Heilig, Cole, & Aguilar, 2010; Means & Knapp, 1999; Phelps 2012). In a study about mathematical attitudes and beliefs, Black and Latino children agreed with statement such as, "there is only one way to solve a math problem" and "learning mathematics is memorizing facts" (Strutchens & Silver, 2000). Thus, children learn to take up a role of rule follower and learn mathematics as a set of static abstract facts, as opposed a process of problem solving, explanation, and justification. Not only are children denied access to the mathematical practices that promote learning, but children's mathematical agency is suppressed from developing.

Children in this context view mathematics as one-way solutions and individually constructed.

Teaching to tests. An expanded narrative of the issue of teaching to tests is warranted as it relates to access to mathematical practices and construction of agency for Latino/a emerging bilingual children. The No Child left Behind (NCLB) act mandates that all children, including Latino/a emerging bilingual children, participate in statewide assessments (U.S. Department of Education, 2016). The issue becomes circular: more and more Latino/a emerging bilingual children in US schools are receiving low scores on state assessments due to low quality instruction; because the focus of instruction is "teaching for a test", test scores continue to decline for this population of children (Menken, 2008, 2010). Thus, the use of high-stakes testing to assess children's math achievement in the classroom is creating more disparities between the most affluent children and low-SES children like Latino/a emerging bilingual children in K-12 schools (Anyon, 1981; Harry & Klinger, 2014; Ladson-Billings, 1997; Means & Knapp, 1990).

This is a growing concern; not only are children marginalized in their ability to receive content knowledge (e.g. procedural knowledge and skills vs. conceptual knowledge) but they are also denied opportunities to explore "mathematical literacy" in instruction. In other words, they are denied the ability to use mathematics "beyond school" and "as a tool to analyze society and solve problems of importance" in their lives (Gutierrez, 2008, p. 360). Mathematical literacy not only helps these children learn how mathematics connects to the real world, but provides children with mathematical agency to choose how to understand and solve problems in a way that makes sense to them, thus promoting their mathematical roles as mathematical agents.

Denial of native language as a resource. In addition to denying Latino/a emerging bilinguals equal access to math assessments, US schools often require that all children use English only in math classrooms. Yet, researchers (Abedi and Lord, 2001; Echevarria, Short & Powers, 2006) have documented the importance of language on math performance of Latino/a emerging bilingual children. Abedi and Lord (2001) investigated how modifications and changes to the English language while keeping the same mathematical concepts in these assessments could change performance. They modified unfamiliar or rare words, changed tense verbs from passive into active, simplified phrases and questions, and made abstract concepts more concrete. Findings from the study revealed significant improvement on performance once the modified version was administered to this population. Further, research has found Latino/a emerging bilingual children's limitations on the use of their language makes them vulnerable to placement in low-track courses and as a result are denied access to mathematical practices (Planas & Civil, 2013)

Unfamiliarity with the context. Some researchers argue that having access to their native language is not enough, it is also important to include familiar contexts that access students' funds of knowledge (Gonzalez, Andrade, Civil & Moll, 2001). Latino/a emerging bilingual children are asked to work with tasks or contexts that are unfamiliar to them or detached from their home or community environments. The tasks become irrelevant, making it difficult or even impossible for these children to make connections between the mathematical ideas in the task to the knowledge they do in fact possess. Instead, tasks could be created to be attached to children's prior knowledge that connect to family and personal experiences as an initial way into familiarity with the context. The personal experiences and connections to family may not be directly linked to children's funds of knowledge but they attach to similar contexts. Quintos, Civil, and Torres (2011) argue that traditional math classrooms teach children as "objects, assuming they do not have any responsibility in the negotiation of the meanings that they learn", and this approach is problematic because children and adults alike learn from making sense of the world, in this case making sense of the math, participating in negotiations of the mathematics, and becoming agents within a community, thus potentially developing mathematical agency (p.237).

Exclusion of home experiences. Some inquiry practices have shown to be successful with children, but it is also noted that these inquiry practices are still detached from home experiences, enough to deny access to Latino/a emerging bilinguals to mathematical practices that promote learning (Ladson-Billings, 1997; Gutstein, 2003). Inquiry spaces in science classrooms are sometimes at fault, and could potentially suppress Latin/a emerging bilinguals' culture and values. Jegede and Aikenhead (1999) warn educators and researchers that if science instruction denies children's cultural world views it could potentially further marginalize and "assimilate" children who are struggling and make them feel like outsiders. Thus, science instruction devoid of cultural views could be seen as potentially harmful for their learning environments. Similarly, Gutstein (2003) explains how engaging in inquiry practices is not enough, that we should teach mathematics to empower children to be agents in their own learning for social change.

Lack of resources. The Latino emerging bilingual population makes up about 80% of all the ELL population in U.S. schools (NCES, 2016). Kozol (2005) showed that schools that serve poor children of color are unlikely to have access to resources such as content books, labs, computers, technology, certified and prepared teachers. In fact, it has been noted by many that "children of color, Latino/a's included, are more segregated than even before" (Ladson-Billings, 1995, p.55). Minority children have limited resources, in particular limited access to the mathematics standards outlined in the Common Core and *Principles of Learning* (Martin & Larnell, 2013; Oakes, 1990; Tate, 1996; 2002; 2008). For such reasons, lack of resources continues to segregate Latino/a emerging bilingual children in US schools, and thus further marginalizes this population preventing them access to the mathematical practices that would help them be successful.

2.4 Barriers to Access Reflected in Performance of Latino/a Emerging Bilingual Children

Barriers to accessing quality mathematics instruction often become evident in lowered mathematics performance. Systemic processes, such as Multi-Tiered Systems of Support, implemented in school districts monitor effectiveness of classroom instruction with universal screeners to measure children's performance (Reed, Weiser, Cummings, & Shaprio, 2012). If lowered performance is sustained by children in the classroom, children are identified as needing additional layers of support to bolster their mathematics performance. Often, the additional supports come in the form of supplemental mathematics instructional interventions.

Supplemental interventions are designed for use alongside whole class mathematics classroom instruction and take place several times a week in small groups of three to six children for 15 to 40 min several times a week over 10 to 20 weeks. Typically, such intervention programs incorporate features of what is often referred to in the special education literature as explicit, systematic instruction (Coyne, Kame'enui, & Carnine, 2011; Gersten et al., 2008; Vaughn, Wanzek, Murray, & Roberts, 2012). Features of explicit, systematic instructional design typically include (a) a focus on big ideas within a specified content; (b) specified strategies by the teacher to be used by children to learn new material; (c) teacher-directed learning (i.e., teacher owned and modeled thinking) when new ideas are introduced, with children's restatement of that thinking practiced; and (d) a purposeful review of previously mastered content (e.g. Brosvic, Dihoff, Epstein, & Cook, 2006; Freeman & Crawford 2008; Freeman 2011; Orosco, 2014).

Pathways to special education and issues of overrepresentation. Latino/a emerging bilingual children who are identified as struggling in the math classes and on state-mandated assessments are usually at a higher risk of being identified as needing services beyond supplemental mathematics instruction; namely, special education services (Sullivan & Ball, 2013). Research on disproportionality reveals an overrepresentation of Latino/a emerging bilingual children in special education, particularly in high incidence categories such as specific learning disability (LD) (Sullivan, 2011; Artiles, Rueda, Salazar, and Higareda, 2005). Latino/a emerging bilingual children become resistant to mathematical instruction because their prior knowledge, language tools, culture and familiar experiences are disconnected from their math classes and therefore are marginalized and oppressed by the pedagogy. Thus, research on mathematics instruction that provides opportunities of access to mathematical practices that allow Latino/a emerging bilingual children use of their prior knowledge, native language and culture, and familiar experiences could be investigated to document how the practices could potentially provide opportunities to exhibit mathematical agency and learning and potentially decrease the disproportionality in special education placement.

2.5 Research on Access to Mathematical Practices for Latino/a Emerging Bilingual Children

This section will discuss studies that have adopted mathematical practices that focus on children's mathematical thinking and understanding with Latino/a emerging bilinguals (Moschkovich, 1999; Turner et al, 2013; Hunt & Empson, 2014). All three articles presented here investigate how Latino/a emerging bilinguals participate in constructivist pedagogies centered on children's mathematical thinking (Carpenter et al., 2015). However, all three articles presented have different research goals, and none focus on building mathematical agency for the subpopulation of Latino/a emerging children who have been identified with a learning disability. In this section, I will discuss each study's goal, conceptualization of mathematical practices, and insights into Latino/a emerging bilingual children who present math difficulties. I will conclude with an analysis of the instructional pedagogies, methodology used, and findings revealed, thus leading to discuss the importance of the current study in addressing the gap in the literature.

Moschkovich's study. Moschkovich (1999) conducted a research study with 33 third grade Latino/a emerging bilinguals participating in mathematical discussions about geometry concepts. Her research questions investigated the kinds of teacher moves that facilitate the participation of Latino/a emerging bilinguals in math discussions and the types of prior knowledge and talk children use to communicate in these discussions. Moschkovich conceptualizes mathematical discussions as "purposeful talk on a mathematical subject" where there are authentic student contributions and interactions (1999, p. 12). This is similar to what Webb et al., (2014) describe in their description of mathematical argumentation and justifications that promote mathematical understanding. The Moschkovich findings reveal that Latino bilingual children benefit from the teacher promoting practices such as "paraphrasing each other's statements", "explaining their methods to others" and from engaging in conversations at length to make sense of "each other's ways of thinking" and more importantly from using their prior knowledge (1999, p.12).

Turner et al. study. Turner et al. (2013) conducted a teaching experiment with 34 fourth and fifth grade Latino/a emerging bilinguals participating in mathematical discussions on ratio and fraction concepts in an afterschool program. Of the 34 children 22 were English learners, who were not all were classified as having math difficulties. Their research questions investigated the kinds of teacher positioning moves that facilitated the participation of seven Latino/a emerging bilinguals in math discussions that produced children as "claim makers, evaluators of ideas, problem solvers" (p.201). Turner and colleagues use mathematical practices as a leverage tool to position children as authorities in agentive mathematical roles. Similar to Moschkovich study, Turner et al., utilizes math argumentation and justifications to promote mathematical understandings. The authors provide pedagogical positioning moves to educators and researchers for engaging children in mathematical practices that promote learning and mathematical agency.

Hunt and Empson's study. Hunt and Empson (2014) conducted individual clinical interviews with 10 third through fifth grade children with LD labels to uncover initial conceptions of fractions in the context of equal sharing word problems. Of the 10 participants five were Latino children with LD identifications. They provide evidence that indicates that the average student comes to understand fraction concepts just as children with LD labels do. Although they are careful not to generalize, it is important to note that these 10 children were equally capable of solving problems as other children with no LD labels.

What has yet to be done? Moschkovich uses discourse analysis to find meaning in how children engage in mathematical discussions, she documents how language can help build their mathematical understanding and more importantly she finds that concentrating on mathematical talk and not on math vocabulary affords Latino/a emerging bilinguals the opportunities to be successful in whole class discussions. From her study, what is unclear is the differences among the 33 children in the 3rd grade classroom. Besides being identified as emerging bilinguals, what else identifies them as mathematical learners? Do some, all, or few struggle in mathematics, are any labeled LD? Are there any differences between these different groups of students when participating in mathematical discussions? Turner et al. use the constant comparison method to delineate positioning roles of seven Latino/a emerging bilingual children when engaged in mathematical discussions around fractions and ratio concepts. What is missing, and beyond the scope of the study, is how do Latino/a emerging bilingual children who have persistent math difficulties engage in mathematical discussions to promote mathematical agency? Hunt and Empson use the constant comparison method to analyze initial conceptions of ten children with identified LD labels when engaged in equal sharing tasks. Their study uncovered initial mathematical conceptions of five Latino children with LD labels, and conducted individual clinical interviews, and, although a very important contribution to the literature, they did not investigate how these children exhibited mathematical agency. Thus, it is essential to investigate how Latino/a emerging bilinguals with persistent math difficulties enact or develop mathematical agency when engaged in mathematical practices.

Chapter 3: Methodology

I will document the pathways by which engaging Latino/a emerging bilingual children identified as struggling in mathematics or LD access mathematical practices and how these practices might support children in developing and enacting *mathematical agency* during a series of problem-solving discussion sessions. I will explore the following research question:

> How do Latino/a emerging bilingual children with learning disabilities and difficulties develop mathematical agency as mathematical learners through their participation in mathematical practices?

3.1 Participants and Setting

The teaching experiment study involved three Latino/a emerging bilingual children in grades three to four from Dominguez Elementary School[1] located in a large urban city in the southern United States. During the 2016-2017 school year (the year of the study) Dominguez Elementary School had approximately 305 students in Kindergarten to fifth grade. The student composition was 7.2 percent African American, 83.6 % Latino/a (primarily of Mexican decent), 1.6 % European American, and 7.5 % Asian. Most the school's Latino/a population primary language was Spanish but many students were accustomed to speaking English on an everyday basis. About 45.9% of the students were classified as English Language Learners (ELLs). Ten and a half percent of students were receiving special education services, and 82.3 % were classified as coming from low income families.

[1] The names of the school, children, and teachers has been changed to protect the identity of the participants.

The sample was purposive; identification of participants was focused on (a) Latino/a children who were identified as English Language Learners, (b) children who had identified math difficulties (may have Tier 2 or 3 identification under the Response to Intervention model) or cognitively defined labels of LD, and (c) an age range of eight to 12 years. Children who had identified math difficulties were those who have been struggling for at least 3 years and designated as needing additional math instructional support in small group settings in or outside the classroom (Orosco, 2014). Children who had identified cognitively defined LD labels are those children who have individualized education goals in math (IEP) and sustained, low performance on math standardized exams. The identification process also included performance measures via the Woodcock Johnson test of achievement and tests of cognitive abilities[2].

The recruitment process for all participants was as follows. First, I supplied the school administration forms to distribute to children who met the criteria. Second, the school administration provided these children a consent form and assent form for their parents and the child to sign. Participants were selected as those who assented and whose parents consented to participate in the study. Finally, since only three students provided informed consent, no priority was given to select students who had identified LD labels. Of the children who consented, all were selected to be participants in the study. Information was collected from the school which included each child's IEP information and Fully Individualized Education (FIE) folder for those with identified LD labels. Information in

[2] Tests of cognitive abilities were: Woodcock-Munoz Pruebas de habilidades cognitivas, 3ra Edición (WM-III Cognitiva); Wechsler Intelligence Scale for Children, Fourth Ed.-Spanish (WISC-IV Spanish); Woodcock-Johnson IV Tests of Cognitive Abilities (WJ-IV Cognitive); Kaufman Assessment Battery for Children, Second Ed. (KABC-II).

the FIE folder included child's ethnicity, gender, and test scores describing the LD identification, such as the Woodcock Johnson Tests of Cognitive Abilities.

3.2 Data Collection

The study consisted of two data collection phases. The first consisted of observations of all participants and their respective teachers who consent to be observed in the math classroom. The second consisted of the instruction of the teaching experiment sessions, teacher reflections and plans, and validity checks of mathematical practices being implemented.

3.2.1 Children's Mathematics Class Observations

The first phase of data collection and analysis consisted of participant observations in their respective mathematics classrooms. Three 30-minute observations of two participants occurred during and after the teaching experiment study. I annotated field notes (Miles, Huberman, & Saldana, 2014) of the participants' engagement and interactions in the math classroom. The purpose of the first phase was to observe the classroom social norms and the participation of each participating child during whole class discussions and small group activities. I also observed how the participating child was positioned in the classroom by the teacher. These observations served as guides to understand each child's participation interactions and to inform my teaching of the sessions.

3.2.2 Teaching Experiment

The second phase of the data collection involved two parts: (a) instruction of the teaching experiment sessions, and (b) teacher reflections about student's participation and plan of future tasks for future sessions. During phase two, twelve teaching experiment sessions occurred. I planned each tutoring session in the teaching experiment with the help of a research assistant and taught each of the lessons.

A teaching experiment was planned for this study. In the constructivist teaching experiment, the researcher acts as the teacher whose role is to introduce mathematical tasks to individual or groups of children (Steffe & Thompson, 2000). A teaching experiment has three components: (a) planned instructional activities by the researcher, (b) a hypothesis of learning process in which the teacher anticipates how children's thinking and participation might evolve when future instructional activities are presented, and (c) collected data sources throughout the instructional sessions to provide evidence of current conceptions of children's thinking and participation. Usually the interest of the teacher is help children learn a mathematical goal, and it becomes important for the teacher, as the researcher, to hypothesize in the moment what the child or group of children might do and find ways to foster this learning (Simon, 1995). Yet, Cobb (2000) noted that children's participation in the experiment can "become legitimate objects of inquiry", such as the created group social norms, what counts as mathematical arguments, and the mathematical practices enacted (p.312). Thus, a teaching experiment can be implemented with mathematical goals in mind, but with the purpose to examine the participation of children in social environments such as a small group discussion (Cobb, 2000). Thus, the teaching experiment methodology was particularly appropriate for this study, because it primarily investigated children's enacted agency when engaged in mathematical discussions, and secondly a hypothesis of the learning was formulated to anticipate how each child's mathematical thinking and participation would evolve on base ten and fraction word problems.

The teaching experiment sessions were leveraged to provide insight on how traditionally marginalized populations who are engaged in mathematical practices might build mathematical agency. Tutoring sessions consisted of small-group problem solving situations and children's mathematical strategy discussions. No teacher directives were given. The primary mathematical content goal of the tutoring sessions was to learn and solidify concepts of base ten and fractional number concepts (Carpenter, Fennema, Franke, Levi, & Empson, 2015; Empson, 2014). The primary mathematical practice goals of the sessions were to engage participants in mathematical practices that might promote the development of mathematical agency (i.e., "making sense of problems", "constructing viable arguments", and "taking ownership of reasoning when critiquing the reasoning of others"). The goals of engaging the children in cognitive demanding tasks about base ten and fractional concepts along with mathematical practices were appropriate to investigate the agency exhibited by children because they provided opportunities for children to have a choice in how to solve word problems and positioned them as competent learners.

Roles. The primary role of the children during the instructional sessions was to (a) attempt to solve the story problems in whatever ways that made sense to them, (b) communicate verbally, in a language that they are more comfortable with, their thought processes of the strategies and mathematical thinking, and (c) ask questions to their peers and the teacher about the problems or to request assistance when needed. My primary role as the teacher was to (a) present the children with appropriate problems that are based on their thinking and an analysis of situations involving base ten and fraction problems, (b) encourage children to build on their informal knowledge of whole number, rational number, and operation concepts, aiding on request or when needed (c) facilitate mathematical discussions among their peers and during whole group discussions about strategies and mathematical ideas employed.

Instruction of sessions. The researcher conducted and taught all teaching experiment sessions. All 12 sessions were video recorded. I utilized a journal to record notes on my thinking following each teaching experiment session. Each child's written work was collected at the conclusion of each teaching experiment session. During the instruction of each of the sessions I used the tasks (see Table 3.3 for description of tasks implemented) and teacher moves (see Table 3.1 for a description of moves), and implemented the problem-solving model (outlined in the next section). I made decisions in the moment about what tasks and teacher moves were appropriate, keeping the primary objective in mind, i.e., to engage participants in mathematical practices that could potentially promote the development of mathematical agency.

Problem Structure. All tutoring sessions were implemented using cognitively demanding tasks (e.g. Henningsen & Stein, 1997), centered on a problem-solving model, having three specific phases (Stein, Engle, Smith, & Hughes, 2008). In the first phase, the launch of the task, I presented a story problem to all children during a whole group session verbally and encouraged them to think aloud and share with others what the problem was about and what it was asking them to do. During the launch phase, I encouraged them to work with each other and to solve it in a way that makes sense to them.

In the second phase, the exploration of the task, all children worked on the problem presented, either individually or in pairs (their choice) in the small groups. As children worked on the problem, they were encouraged to explain their strategies and ideas with me and their peers. I listened attentively to their mathematical ideas and their verbal conversations with their peers, supporting them to provide complex explanations, clarifications, elaborations, and justifications of the strategies used. I made sure to attend to the details of their problem-solving strategies.

In the third phase, the discussion phase, all children were gathered together forming a circle where everyone was visible and ready to actively listen to some of the strategies used by children during the exploration phase. I encouraged several children to share their strategies and for their peers to actively listen and provide feedback. During the discussion phase, I executed several teaching moves. The teaching moves included: ensuring the children were making sense of the problem, clarifying and eliciting children's thinking, assigning competence, and extending their thinking (see Table 3.1 for a description of these moves) (Jacobs & Empson, 2016; Turner et al., 2013).

Table 3.1: Description of Teacher Moves.

Teacher Moves	Description
Ensuring children are making sense of the problem	The teacher will aid children to familiarize themselves with the story context. During the launch of the task, the teacher will ask children to explain what the problem is about. The teacher could ask specific children to describe specific details they know about the story problem (e.g. How many cookies does Maria have? And “How many cookies are inside each box?”) and what the essential question is asking them to find.
Clarifying children’s thinking	The teacher will aid children to explain the strategies used and provide prompts to struggling children to help in clarifying what the problem is about and ask questions to help children link the story problem and the details of their current strategies.
Eliciting mathematical thinking	The teacher will invite individual or pairs of children to explain the strategies used and attend to the details of the strategies used (e.g. I saw that you added five each time, why did you do that? How did that help you?)
Assigning competence to children’s ideas	The teacher will re-voice children’s mathematical strategies and thinking, prompts children to justify their agreement with a peer’s strategy, and invite children to evaluate their disagreement with a peer’s strategy during the explore and discussion phase.
Extending children’s thinking	The teacher will solicit different strategies, ask children to use a number sentence, or ask follow-up problems with challenging numbers during the explore and discussion phase.

To ensure mathematical practices were present throughout all sessions, I kept track of norms, teacher moves, competence prompts by keeping a list with me at all times (see Table 3.2 for an example of the checklist)

Table 3.2: Example of norms and teacher moves checklist.

Norms and Teacher Moves
Solve problems in any way that makes sense to you! o Resuelve el problema de una manera en la cual tú puedas entender
Students can choose to work alone, or in pairs o Pueden resolver el problema en pares o individualmente.
Ask students to listen actively to each other o Hay que escuchar unos a los otros de las ideas que tienen
Agree/disagree with one another respectfully o Respetosamente les voy a pedir que digan si están de acuerdo o no con las ideas de otros, y por qué.
Explain why they believe something, in any language they wish o Y también que expliquen porque creen en tus ideas y las ideas de otros
Convince everyone of their statements o Por último, les voy a pedir que nos convenzan con las ideas que generen.
Teacher prompting for clarification of the child explaining a strategy, and accept and build on prior knowledge o ¿Haber explícame que hiciste? ¿Cuál es tu estrategia? A ya entiendo, ¿Y porque hiciste 6 mas 6 y no 10 mas 10?
Teacher re-voicing student's comments to position as an expert among peers o Mario en su estrategia hiso 10 mas 10, a ver Jorge y Lisa están escuchando lo que hico Mario.
Teacher will promote student re-voicing other student's comments o Jorge me puedes explicar que fue lo que dijo Mario para su estrategia
Teacher re-voicing student's comment to validate strategies used. o Juan veo que pusiste 5 en cada caja, ok, ya entiendo lo que hiciste. Entiendo muy bien tu estrategia.
Teacher will prompt the justification of why they think this strategy is correct o Lisa veo que tú estás de acuerdo con las ideas de Mario, me puedes explicar porque estás de acuerdo. ¿Alguien más está de acuerdo con Mario? ¿Juan tú me puedes explicar porque estás de acuerdo?
Teacher will prompt children to validate ideas presented- to position as evaluators of math ideas. o Mario creo que tú no estás de acuerdo con las ideas de Juan, me puedes explicar porque no estás de acuerdo.
Teacher will extend children's thinking o Lisa ya entiendo tu estrategia, ok, si te cambio los números a 10 y 4, ¿como resolverías el problema?

Planned tasks. The mathematical content of the teaching experiment sessions began with addition and subtraction story problems to explore initial conceptions of number. Following addition and subtraction story problems, further sessions introduced base ten and equal sharing story problems (Carpenter, Fennema, Franke, Levi, & Empson, 2015). I began with an initial sequence of problem tasks (see Table 3.3 for a description of tasks). The initial sequence was modified to be adaptable to each child's current conceptions (e.g. different number combinations) during instruction, which were also presented in familiar and realistic contexts. I began with a Join Change Unknown story problem, to provide children opportunities to create their own strategies as opposed to a standard algorithm with no connections to the context. Contexts were related to children's personal interests and prior experiences. The problem-solving tasks were designed to be dynamic (i.e., could be solved in a variety of ways using a variety of strategies) to help all children use their informal and evolving mathematical reasoning. Problem tasks were written on a whiteboard and presented in both English and Spanish.

Table 3.3: Example of Sequence of Planned Tasks.

Session	Planned Task Type	English	Spanish
1	**Join Change Unknown**	Jasper has 4 carrots. His friends gave him some more carrots. Now Jasper has 12 carrots. How many carrots did Jasper's friends gave him?	Jasper tiene 4 zanahorias. Sus amigos le dieron más zanahorias. Ahora Jasper tiene 12 zanahorias. ¿Cuántas zanahorias le dieron los amigos de Jasper?
2	**Base Ten Multiplication**	Julia got 3 boxes of cookies. Each box has 10 cookies in it. How many cookies does Julia get altogether?	Julia tiene 3 cajas de galletas. Cada caja tiene 10 galletas en ella. ¿Cuántas galletas tiene Julia en total?
3	**Base Ten Measurement Division** (Groups of 10)	Gorge has 32 toys. He puts 10 toys inside each bin. How many bins can he fill?	Gorge tiene 32 juguetes. El pone 10 juguetes dentro de cada caja. ¿Cuántas cajas puede llenar Gorge?
8	**Equal Sharing**	In Mario's store, there are 8 chocolate bars. Three kids want to share the chocolate bars so that everyone gets the same amount. How much chocolate can each child get?	En la tienda de Mario hay 8 barras de chocolate. 3 niños quieren compartir las barras de chocolate y todos quieren tener la misma cantidad. ¿Cuánto chocolate recibe cada niño?

Teacher reflections and plans. After the conclusion of each of the sessions I reflected on the overall participation and agency of children and the strategies used. I annotated descriptions of children's participation with observations that came to mind, and the prompts and teacher moves that seemed helpful in promoting enactment of mathematical agency and mathematical thinking. Finally, I made plans about the type of task, teacher moves and any decisions that could help promote mathematical learning and agency among the participants for the next session.

Data sources. In the teaching experiment, data collection was facilitated through 12 tutoring sessions. Collection of data occurred within a six-and-a-half-week period, with tutoring occurring one or two times a week in an after-school setting equipped with large tables, manipulative materials (i.e. unfix cubes), journals, paper, markers, pencils, and a whiteboard. Each tutoring session lasted approximately one-hour and utilized small group instruction to analyze situations in which Latino/a emerging bilingual children might develop or enact mathematical agency when participating in mathematical discussion in problem solving situations. Thus, the process yielded three primary data sources: video recordings of the sessions, field notes, and children's written work.

3.3 Data Analysis

Data analysis consisted of four phases: (1) pilot data framework hypothesis for the agency exhibited (2) ongoing analysis of the teaching experiment sessions (3) constant comparison method analysis and classical content analysis of the three embedded case studies within the teaching experiment and (4) coding validity and reliability. Each of the phases is described below.

3.3.1 Pilot Study

This section will discuss preliminary results from a pilot study. The pilot study produced a participation framework for agency exhibited. This work was used as a deductive tool to analyze the data for the agency exhibited by the three emerging bilingual children in the current study.

Pilot participants and setting. A pilot study took place to document participation in the mathematical practices for seven Latino/a emerging bilingual children identified as struggling in mathematics. Data collection took place in an afterschool tutoring program and consisted of seven 45-minute sessions over a four and one-half week period. Two teachers who are experts on children's mathematical thinking and the use of problem solving to develop base ten concepts taught the lessons.

The participants consisted of five Latina females and two Latino male children with ages ranging from eight to nine years old. All children came from low-socio economic backgrounds with parents of Mexican nationality, had been identified as struggling children since 2nd grade, and knew both Spanish and English. All children were receiving additional mathematics instruction in or outside their math classroom. None of the children had identified LD labels. All children had initial conceptions of addition and subtraction operations, and had whole number understandings, based on interactions from our first session. The range on base ten concepts varied among all children.

Task trajectory. I designed an initial sequence of problem tasks and possible teaching moves (see Table 3.1 for teaching moves). Tasks were dynamic (i.e. adaptable to each child's current conceptions), situated in number operations and base ten situations (see Table 3.4 for pilot tasks) and presented in familiar and realistic contexts. The problem-solving tasks were designed to be dynamic (i.e., could be solved in a variety of ways using a variety of strategies) to help all children use their informal and evolving mathematical reasoning to come to a solution.

Table 3.4: Pilot Study Tasks.

Session	Task
1	**Join Result Unknown** Amelia has 13 jellybeans. Her brother gives her 8 more jellybeans. How many jellybeans does Amelia have now?
2	**Join Change Unknown** Jose has 5 chocolate bars. His friends gave him some more chocolate bars from their Halloween Candy. Now Jose has 12 chocolate bars. How many chocolate bars did Jose's friends gave him?
3	**Base Ten Multiplication** Jasper got 4 boxes of creepy carrots. Each box has 10 carrots in it. How many carrots does Jasper get altogether?
4	**Base Ten Measurement Division** (Groups of 10) Alondra has 64 stickers. She pastes them in her sticker book so that there are 10 stickers on each page. How many pages can she fill?
5	**Multiplication, multistep** (Groups of 10) Yoselin has 11 packages of cookies. Each has 10 cookies in it. She also has 6 extra cookies. How many cookies does she have in all?
6	**Multiplication, multistep** (Groups of 10) Fatima saw 14 space ships outside her window. Each spaceship has 10 aliens in it. She also saw 8 extra aliens walking by her house. How many aliens did she see in all?
7	**Multiplication** Juan's grandma has 6 bags of tamales. There are 12 tamales in each bag. How many tamales does Juan's grandma have all together?

The teachers and I reasoned that instruction might begin with tasks that attach to children's initial understandings of addition and subtraction problems, inviting children to use whole number concepts and directly modeling strategies (e.g. drawing all objects or modeling using the action on the problem). Then we transitioned to tasks with base ten concepts.

The mathematical tasks were presented using the teaching moves within the 3-phase problem-solving model described earlier (Stein et al., 2008). The teaching moves during the problem-solving model were planned to (a) promote discourse interactions among all children, (b) provide competence while supporting each child in their mathematical conceptions, and (c) document the mathematical agency exhibited during problem solving and discussions.

Analysis of pilot. The data presented focuses on pilot data from all teaching sessions, examining the relationship among the children's mathematical decisions and participation actions during the tutoring sessions, delineating mathematical agency patterns observed. In this section I discuss the preliminary results from the pilot study and how they provided the agency framework for my initial coding.

After transcribing and summarizing the audio taped math discussion sessions and expanding upon field notes and reflexive journal entries, I conducted a general read-through of the data (Corbin & Strauss, 2008), paying attention to the whole body of data and making analytic memos about the insights, patterns, and possible themes and questions that came to mind. Transcripts of the children's participation during sessions were read in their entirety to capture overall themes. Children's work, small group video, teacher small group reflections, and my field notes were used to add context to the mathematical discussion sessions.

For the first past of the data, I coded for instructional interactions, particularly those where child to child or teacher to child discursive interactions occurred. Drawing from Forman and Ansell (2001), I defined an episode to be a coherent instructional interaction where an entire discussion occurred around a single strategy, either constructed by one child or a pair of children, for solving a problem. I attended to verbalization and gestures by teachers and children to communicate ideas. I kept original Spanish and English transcriptions for data analysis, as I am a native Spanish speaker. To present results later I translated Spanish sections into English with careful consideration to maintain meaning and interpretations of what was said. I repeated this process several times, which resulted in the identification of 38 episodes which I analyzed further.

Next, I performed a constant comparative approach to delineate observable patterns of children's participation patterns and agentive roles while engaging in mathematical discussions of each episode (Glaser & Strauss, 1967). I began with assigning open codes to episodes to construct categories pertaining to mathematical agency by examining (a) ways in which individual or pairs of children shared strategies with others during the exploration and discussion and (b) the interaction of individual children attending to the mathematical ideas and thinking of others, also, noting how the participation and engagement of individuals shifted over the course of the sessions. The primary purpose was to capture any shifts that could occur as the sessions progressed. After assigning codes to the episodes of the data I began to construct categories by grouping open codes, thus using axial coding (Corbin & Strauss, 2008) to identify common themes and categories (e.g. open codes "sharing procedures for strategies" and "mimicking what others do to serve as what counts as math" to a category of "un-original ideas"). In the selective coding phase, an agency framework of enacted participation during the problem solving and discussion phases (see Table 3.5) emerged.

Table 3.5: Enacted participation and agency during the problem-solving and discussion phase.

Degrees of Mathematical Agency	**Codes describing the actions in which children were engaged**

(Limited)	• Sharing un-original ideas
	  • Sharing procedures for strategies
	  • Using the strategies of others verbatim
	• Refusal to participate with peers or teacher

Elicited Participation **(Developing)**	• Sharing original ideas
	  • Sharing mathematical thinking
	  • Sharing problem solving strategies
	• Somewhat hesitant at times and wanting to share ideas

Un-elicited participation **(Enacting)**	• Unprompted contributions to share with others
	  • To defend mathematical ideas
	  • To argue mathematical ideas
	  • To ask for clarification
	  • To share math thinking of original ideas

Agency framework from pilot study for dissertation study. In analyzing the pilot data, I found several kinds of actions in which the seven Latina/o emerging bilingual children struggling in math that describe their agency exhibited (see Table 3.5). These actions included children participating in sharing their thinking of original and un-original strategies and take an action on their mathematical ideas. I noticed three types of agency across the sessions over time, that showed how agency became richer as children engaged in meaningful discussions about base ten concepts. Children began to create their own strategies abandoning standard algorithms. They also began to evidence an understanding of how their strategies were solved, reflecting on their thinking and sometimes sharing their math thinking when prompted by the teacher. As the sessions continued, I noticed changes in how they engaged in conversations with peers and the teachers, were they began to want to participate regardless of the teacher prompting or eliciting their thinking.

This framework supported the continuation of a similar analysis of the dissertation study in that it provided preliminary findings on how similar aged Latino/a emerging children would participate in mathematical discussions. Both groups of participants, in the pilot study and dissertation study had similar demographics, family backgrounds (most of Mexican decent), and were struggling in math. In addition, the two teachers in the pilot study utilized the mathematical practices in the teaching of the sessions that I, as the teacher, was about to employ during the enactment of the dissertation study. Because the children and the teachers had similar characteristics, I found this framework to be appropriate in the analysis of future data with three emerging bilingual children with identified LD or struggling labels.

3.3.2 Ongoing Analysis.

During the collection of the teaching experiment sessions, I and a graduate student researcher conducted ongoing analysis before and after each session to uncover ways in which the three participating children exhibited agency and shared their mathematical thinking (Simon et al., 2010). After the conclusion of each session, I and the graduate student discussed what transpired in the session pertaining to (1) critical shifts in children's participation seen as agency and (2) the mathematical strategies used. At times the graduate student was present in doing observations of the sessions, but most often would watch the videos of the sessions after the conclusion of my teaching. We used what we learned to plan future sessions to promote the learning and engagement in discussions about base ten and fractions, paying close attention to the presence of instances that indicated a form of math agency, participation, engagement and mathematical thinking and strategies used by each participating child in the study.

3.3.3 Embedded Case Studies.

During the collection of the teaching experiment data, I used embedded case studies (Yin, 2009) as the main form of analysis for how agency was exhibited by the three children participating in the problem-solving discussions. The main unit of analysis was the mathematical practices implemented in the study; and the three children's enactment of agency were the embedded case studies within the teaching experiment. Stake (2005) makes a clear distinction between what he calls a nested case study, also an embedded case study, and a multiple case study in that it "gains its integrity from the wholeness". In other words, the individual cases (the three emerging bilingual children) served to explain the phenomena occurring in the main case study, the mathematical practices present in a teaching experiment of problem solving, which are part of and also integral to each of the individual cases (p.153).

Constant Comparison Analysis. The three embedded case studies within the teaching experiment were analyzed using constant comparison analysis (Glass & Strauss, 1967). Constant comparison analysis was appropriate for the analysis of the three case studies because it can be used "deductively (e.g. codes can be identified prior to analysis and then looked for in the data) inductively (e.g. codes emerge from the data) or abductively (e.g. codes emerge iteratively)" (Leech & Onwuegbuzie, 2007). I used constant comparison of the teaching experiment data to identify how agency was exhibited based on the hypothesis codes from the constant comparison method obtained during the pilot data by examining the interactions of each case study across all the sessions.

Before beginning to analyze the data, I identified session episodes (n=111) using MAXQDA to delineate patterns of interactions and participation among participants. Episodes were identified as coherent interactions of the participants or the teacher and participants around a single mathematical strategy in the exploration and discussion phase of each of the sessions conducted (Corbin & Strauss, 2008). I attended to the verbal communication, gestures, and utterances that occurred in each episode. Once episodes were identified, I documented memo notes of each episode answering questions to find out when, who, what and how children explained and showed their mathematical thinking with others and note any changes across the teaching sessions. Then, I transcribed the episodes to further expand and highlight agency patterns and non-verbal cues. Then, I began to perform constant comparison analysis to help answer the research question after all data had been collected. Data analysis included a triangulation of video recordings of every session, MAXQDA episodes, memo notes, transcripts of episodes, student work, and journal entry notes.

During the constant comparison analysis for the data I used the pilot study framework as a deductive tool to analyze data episodes with hypothesis codes. Hypothesis coding is the "application of a researcher generated list of codes onto qualitative data to ask a researcher-generated hypothesis" (Miles, Huberman, & Saldana, 2014). Because the pilot data framework was used as a deductive tool, this method was appropriate for a continuation of constant comparison analysis of my qualitative data set (Leech & Onwuegbuzie, 2007). Thus, the pilot data framework served as a predetermined list of codes to analyze the qualitative data from each of the episodes to construct categories pertaining to mathematical agency by examining (a) ways in which individual or pairs of children show limited, developing or enacted mathematical agency, and (b) ways in which they began developing mathematical agency. Also, noting previous hypothesis coding analysis, if the codes did not fit, or portrayed negative cases, I continued to examine (c) ways in which individual or pairs of children shared strategies with others during the exploration and discussion and (d) the interaction of individual children attending to the mathematical ideas and thinking of others.

Next, constant comparison analysis of the qualitative data set continued, with the purpose to refine the hypothesis codes into categories. Categories were refined to finalize emerging themes in the outlined episodes. Emerging themes pertaining to the observable patterns of children's participation and enactments of mathematical agency while engaging in mathematical discussions and problem solving situations continued to be refined by comparing previous codes with new chunks of data. I took similar coded chunks and grouped them together and labeled them with the same code. For example, similar coded categories in regards to type of agency exhibited such as "explaining own solution and not that of their peers" and "evaluating the idea of a peer by simply agreeing with no explanation as to why" were grouped together as "developing math agency" and codes such as "evaluating the idea of a peer" and "explaining the solution of a peer" were coded together and labeled as "enacting math agency". Whereas codes that seemed dissimilar were placed in different categories such as "children are not making sense of their mathematical strategies" and "children are attempting to make sense of a standard algorithm strategy used", the former as "low math agency" and the latter as "developing mathematical agency". The iterative process of coding, comparing, refining continued until I had exhausted and coded all the data, thus yielding three different types of mathematical agency, limited, developing and enacting (See Table 3.6 for agency code and definitions).

Table 3.6: Mathematical Agency Definitions.

Degree of Mathematical Agency Codes	Code Definitions
Limited Math agency	Low math agency refers to a child or group of children using procedures without making sense of the mathematics and any actions that might indicate a refusal to participate in sense making. A child may also be hesitant and refuse to take ownership of their mathematical ideas/strategies by not explaining or justifying their thinking. When listening to peer's mathematical ideas, they will not engage in discussions to accept, justify or argue their peers' ideas.
Developing Math agency	Developing math agency refers to a child or group of children's capacities to makes sense of the mathematics, and any actions that might indicate participation in sense making. When asked to share mathematical ideas/strategies a child may begin to take ownership by explaining their thinking. When listening to peer's mathematical ideas, a child will most likely not engage in discussions to take up, justify or argue their peers' ideas.
Enacting math agency	Enacting math agency refers to a child or group of children's capacities to makes sense of the mathematics, and any actions that might indicate participation in sense making. When sharing mathematical ideas or strategies, a child will take ownership by explaining or defending their thinking. When listening to peer's mathematical ideas/strategies, a child will most likely engage in discussions to take up, justify or argue their peers' ideas.

Classical Content Analysis. After conducting constant comparison analysis of all the data and delineating three mathematical agency codes and their respective definitions I was interested in delineating times of occurrence of each of these agency constructs across the sessions to see if shifts had occurred for each of the case studies. I engaged in classical content analysis to uncover each child's mathematical agency exhibited across the sessions (Leech & Onwuegbuzie, 2007).

I and the research assistant engaged in analyzing each of the episodes uncovered previously. We began to look at the data and coded for each child participating in each of the episodes (n=111) and coded for either limited, developing, and enacting agency. At times when it was impossible to categorize the agency exhibited by a child in a single episode, that episode was discarded as evidence. For example, if a child was not part of the participation (e.g. no discourse or actions) or engagement in a conversion with others but mainly served as a bystander observing two peers or a peer and the teacher, we decided we did not have enough evidence to assign an agency code to that child in that episode. The research assistant and I met to code the first session together, noting and coding for limited, developing and enacting agency. After the conclusion of the first session, we coded the remaining of the sessions individually for agency.

Triangulation of Data. Video session collected as MAXQDA episodes and transcripts of episodes from the teaching experiment study were analyzed and coded by looking for instances in which children(s) problem solving was explored and discussed for agency as either limited, developing or enacting. Field notes and memo notes were used to corroborate the explanations of their math strategies and agency findings from the episodes. Artifacts of the tutoring sessions such as student work, and teacher-researcher reflection journal entry notes were used as further evidence of the session analysis.

3.3.4 Coding Reliability.

I selected all 12 teaching experiment sessions conducted in the study, encompassing 100% of the data. The graduate research assistant with relevant research knowledge independently coded the sample data. After coding for degrees of agency, we met to establish inter rater agreement (IRR) using (agreements / (agreements and disagreements) across all the episodes outlined throughout the entire data set of each teaching experiment. We met after each teaching experiment session and coded each time for IRR across all the identified episodes resulting in 80.79 % agreement on the mathematical agency codes of all 12 teaching experiment sessions. After we discussed and clarified any disagreements of mathematical agency codes by utilizing the triangulation of other data sources (e.g looking at student work and journal entry notes, as well as levels of robustness) agreement increased to an overall average of 98.75 % for children's mathematical agency of all 12 teaching experiment sessions.

Validity checks of the mathematical teacher moves. Additionally, validity checks were performed for the first two sessions to check for the presence of teacher moves (see Appendix A for a reference to the teacher moves) using MAXQDA software. The purpose of the validity checks of the teacher moves was to ensure that children were engaged in the mathematical practices described in the research methods and throughout the study. After the conclusion of the first session, a graduate research assistant and I analyzed the entire video of the first session using MAXQDA. We coded individually for these moves and met to confirm or disconfirm the presence of the teaching moves (e.g. assign as expert among the group, extend children's thinking). We coded for instances where the mathematical teaching moves were present with one's, and not present with zero's, and took the total of (sum of the number of instances that the practices were present

/ total number of practices). We conducted a validity check one more time, after the conclusion of the second session, and met to confirm or disconfirm the presence of the teaching moves and peer debriefed for agreement. At the conclusion of the teaching sessions the research assistant conducted two more validity checks selected at random, where 20% of the of the remaining sessions were coded. Agreement resulted in 91.75% of the four coded sessions.

3.4 SUMMARY

This study provided various amounts of data in the form of observations, videos, transcripts, student work and journal entry reflections. The teaching experiment study group provided the opportunity to look at ways in which emerging bilingual children with identified learning disabilities or difficulties participated, engaged in mathematical agency that focused on engaging children in problem solving and how these shifts occurred. I conducted analysis of the pilot data using the constant comparison method analysis inductively, and for the embedded case studies I used constant comparison analysis deductively to create framework for mathematical agency described as limited, developing and enacting. Finally, a research assistant and I conducted classical content analysis to uncover the number of instances in which each case study child exhibited limited, developing, and enacting agency by coding all the episodes present in each of the sessions. These findings are presented in the next chapter.

Chapter 4: Findings

In this section, I present narratives of each child's participation and building of agency in a seven-week long session teaching experiment. Each narrative consists of:

- An introduction to each child and the knowledge she or he brought forward into the teaching experiment (i.e., family background, interests, competencies, classroom instruction).
- The advancement of agency over time in the teaching experiment, presented as each child's individual construction of mathematical agency and at times evidenced through shared spaces between children in the teaching experiment session.
- An overview of the agency patterns across the children and factors that seemed to advance or constrain agency as it advanced in the shared space.

In presenting these narratives, I unpack how Latino/a emerging bilingual children with learning disabilities and difficulties develop mathematical agency as mathematical learners through their participation in mathematical practices. I have also included factors that seemed to have limited the agency of all children during the sessions. Before I introduce each child's narrative, I would like to state my narrative and positionality and discuss why this work is important.

4.1 My narrative as a Latina Emerging Bilingual and positionality

My high school didn't have enough math and science textbooks, so I shared them with other classes. This was a minor inconvenience compared to having a substitute permanently take over my math class during the middle of the year. At first the lack of accountability seemed fun, but it later became apparent that little learning was happening in our class. Despite these resource constraints, I do remember one exceptional teacher, Mrs. B, who made an impact in my schooling. She stayed after school for many afternoons and taught us calculus. Her goal was for us to understand how derivatives and integrals worked. Her class was challenging but in the end my hard work payed off when I aced the AB Calculus AP test.

I excelled in high school and graduated as the salutatorian. However, I struggled immensely during my first two years in college. At the time, I was not aware of my disadvantage, but soon I came to understand how much better prepared my peers were, especially in math and science courses. I worked twice as hard just to catch up to their level of understanding. I remember feeling angry that my prior educational experiences had not prepared me for this challenge and I decided that I wanted to change this by helping my community. I was determined, so upon graduation, I began to teach mathematics at a culturally and linguistically diverse middle school, like Mrs. B serving a majority of low income minority Latina/o population.

My aim was to provide these students with opportunities to learn mathematics content deeply. During my five years as a middle school teacher, I would work after school helping students that were struggling to succeed in understanding threshold concepts in Algebra and Geometry. As my role expanded to math department head my responsibilities increased and I began to mentor other teachers. I wanted teachers to be successful at facilitating mathematics instruction for low income Latina/o students. Teaching students and leading teachers at my middle school was rewarding, yet I felt that perhaps my impact could be greater. Therefore, I decided to pursue a doctorate degree, in mathematics education with the goal to influence a much larger group of underrepresented students.

While teaching middle school mathematics, I became interested in my students' mathematical reasoning and participation during class discussions. One of my students, Armando, a Latino emerging bilingual student, identified himself as "not good at math". When I would ask Armando how he solved a math problem his response usually began with "I don't know if this is correct, but I think it is…". I noticed that these types of responses were common for students who identified themselves as "not good at math", thus not seeing their potential as mathematical agents in their own learning. As I continued to engage with students in math discussions, it became apparent that only one type of student, who I came to think of as "the believers", who categorized themselves as "I am good at math", were the ones who most participated in group discussions. Furthermore, I realized the non-believers were students that tended to struggle because of the English language barrier. I encouraged Armando to talk in Spanish with his peers about the mathematical ideas he had, but it did not seem to be enough. This intrigued me because I realize that it was not enough to speak the language to help Armando understand the mathematical concepts. I wish Armando's story had a fairy tale ending but that was not necessarily the case. These experiences sparked questions in my pedagogy as to how I could help Armando and similar students engage in conversations regarding their mathematical justifications.

At the time, as a middle school teacher, I didn't have the adequate tools to help kids like Armando, however in my current role, as a doctoral student, I realize that there may be ways to help this group of children. As a graduate research assistant, I had the opportunity to work with a professor on her NSF grant about fractions with children who have LD labels. During this time, I came to the realization that the label in special education is mostly due to social factors. I also began to research Latino children with LD labels and I realized then that there exists a disproportionality of Latino children with English

Language Learning labels in special education. This disturbed me. I wondered if any research in mathematics on intervention existed for these children. I realize little research exists, thus I saw it as a need that I could potentially contribute to.

4.2 Emerging Bilingual Children's Stories: Introducing the Narratives

Julia, Martin, and Gabriel were three lively Latina/o children who were emerging bilinguals and had been identified by their school system as having a LD or struggling in mathematics. Before initiating the teaching experiment sessions, I conducted a pre-session without the cameras to get to know Julia, Martin and Gabriel. During the pre-session, I asked them to decorate their math journals with numbers about themselves using markers and crayons. I brought an example of my journal, with numbers about myself where I shared my age, the number and names of most of my family members, my favorite foods, and personal interests (e.g. traveling, running). The purpose of bringing an example of my mathematical journal served to help mediate a casual conversation with all children and create an atmosphere of trust and amicability.

In addition to decorating their journals, I read them a Spanish story from a book named *Las Zanahorias Maleficas*, which translates to *Creepy Carrots*. I purposely chose a Spanish story because I wanted to establish a space where they could use their primary language freely. The pre-session was essential in getting to know each child a little bit more and helping gain each child's trust and acceptance. Through these informal interactions, I eased each child's anxieties about accustomed traditional tutoring settings (e.g., expectations to complete tests, worksheets and practice problems). The pre-session provided a safe space where these children felt comfortable being themselves. The following section describes each child's narrative, their primary language and preference, and the school's positioning of each child.

Julia's narrative. Julia was a Latina emerging bilingual, third grade 9-year-old Mexican American girl, who lived with her mother and brother. Julia also explained that she had a dog named Tody. In her journal, she described her favorite desserts: ice cream and cupcakes, her favorite holidays (i.e., Halloween and Easter), and that she would like to visit Disney World one day (see Figure 4.1). Her favorite activities included swimming and hanging out with family and friends for cookouts. Julia enjoyed school activities and spoke highly of all her teachers. Julia's favorite subject in school was science yet she explained she also enjoyed mathematics.

Figure 4.1 Julia's Journal Drawings during the Pre-session

In brief interactions with me, Julia's mom, a monolingual Spanish speaker, expressed how she prioritized Julia's schooling to include instruction in both languages, Spanish and English, throughout all her classes. Initial interactions with the assistant principal portrayed Julia to be a "sweet child but with many learning difficulties with her mathematics and Language Art classes". The assistant principal suspected that Julia may have had learning difficulties due to a language barrier and issues with reading comprehension. As I learned about Julia's dispositions with language, I learned that she spoke both English and Spanish but predominately spoke Spanish in her home and was learning to read and write in English in all her classes. Throughout my interactions with Julia, she expressed that she struggled to read in Spanish more so than in English, and preferred to read and write in English.

At the time of the implementation of the teaching experiments, Julia was in the process of being referred for special education services. According to the school, Julia was identified as a Tier 3 student under the Response to Intervention (RTI) program (Fuchs et al., 2012). Under this model, Julia was given extra time to complete assignments, provided with assistance in math class, and positioned as needing extra instructional help from teachers on a one-on-one basis in the form of explicated mathematical procedures.

Martin's narrative. Martin was a Latino emerging bilingual, fourth grade 10-year-old Mexican American boy who lived with his mother and had two brothers and two sisters. Martin explained that his dad was in Mexico and had been trying to obtain a residency to come live with Martin and his family in the United States. Martin told me that he loved animals and had a small bunny as a pet. In his journal, he described his favorite foods as pizza and hamburgers, enjoyed spending time with his two uncles, and watching Pokémon with his brothers (see Figure 4.2). Martin stated several times how his life was hard and that he wished his dad was around; Martin shared that he often took care of his little brothers and sisters because he was the oldest child. He also expressed how his mother worked twice as hard because his dad was not around to help financially.

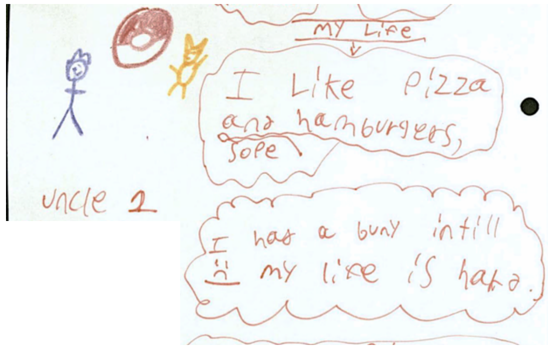

Figure 4.2 Martin's Journal Drawings during the Pre-session

Martin spoke both English and Spanish in school and at home. The school reported that most instruction from K-2 grades was received in Spanish and was receiving instruction in both languages beginning in 3rd grade. Martin had no preference in reading and writing in both English and Spanish. Martin enjoyed school activities and liked most subjects. Particularly, Martin did not express disliking or liking mathematics. Martin was transferred from Dominguez Elementary School, and had been attending Button Elementary since first grade. According to the assistant principal, Martin's teachers portrayed his mathematical performance as below grade level, especially subtraction and multiplication calculations (e.g. regrouping), and that he struggled with completing grade level word problems when following a series of steps in order to obtain a correct answer.

Martin was identified by the school as needing special education services in August of 2016. He was identified with a specific learning disability label in the academic areas of listening comprehension, oral expression, reading fluency, reading comprehension, written expression, math calculation and math problem solving. Martin's cognitive abilities and processes were assessed using the Woodcock-Johnson IV tests of Cognitive Abilities (WJ-IV Cognitive), the Kaufman Assessment Battery for Children, second edition (KABC-II) (see Table 4.1 for assessment results).

Table 4.1: Martin's cognitive ability processes score and rank from WJ-IV Cognitive & KABC-II.

Cognitive Clusters	**Description of Cluster**	**Standard Score**	**Percentile Rank**
Crystallized Intelligence	Breath/depth of cultural knowledge, ability to communicate and reason verbally	78	7
Fluid Reasoning	Ability to reason, form concepts & solve problems, basic reasoning processes	93	32
Short Term Memory	Ability to learn and hold information, then use it within a few seconds	80	9
Long Term Retrieval	Ability to store information and retrieve it, associative storage and retrieval	86	18
Visual Processing	Ability to perceive, analyze, synthesize and think with visual patterns and store/recall	100	50
Processing Speed	Ability to perform automatic cognitive tasks under time pressure- attention and speed	96	40
Auditory Processing	Ability to analyze, synthesize & discriminate auditory information- not comprehension	100	50

Gabriel's narrative. Gabriel was a Latino emerging bilingual, fourth grade 10-year-old Mexican American boy who lived with his mother and nine brothers and sisters. Gabriel's father was estranged from his family since Gabriel was one year of age. Gabriel enjoyed playing sports (soccer, specifically) and conveyed interest in playing video games with friends. In his journal, Gabriel described his favorite foods to be pizza and tamales (see Figure 4.3). Throughout the study, the child expressed his discontent with school personnel and family members. He was not particularly fond of academics (e.g., learning math), preferring engagement in recreational sports at school.

Figure 4.3 Gabriel's Journal Drawings during the Pre-session

Gabriel spoke both English and Spanish yet predominately spoke Spanish at home. Most instruction in his classes occurred in English. Gabriel expressed his preference to communicate ideas mostly in Spanish, and voiced that he sometimes struggled to express himself in English. According to the assistant principal, his teachers reported only being able to communicate in English through one- or two-word phrases. Gabriel's math teacher expressed concerns with his negative behavior. In their view, his constant disruptive behaviors resulted in failure to complete assignments and distracted his peers from mathematics instruction.

Gabriel was identified by the school as needing special education services in December of 2015. He was identified with both a specific learning disability and Emotional Disturbance (ED) labels. He was identified with a specific learning disability label in the academic areas of reading fluency, reading comprehension, written expression, math calculation and math problem solving. Gabriel's cognitive abilities and processes were assed using the Woodcock-Munoz Pruebas de habilidades cognitivas, Third Edition (WM-III) and the Wechsler Intelligence Scale for children, Fourth Edition-Spanish (WISC-IV Spanish) (see Table 4.2 for assessment results).

Table 4.2: Gabriel's cognitive ability processes score and rank from WJ-IV Spanish & WM-III.

Cognitive Clusters	**Description of Cluster**	**Standard Score**	**Percentile Rank**
Crystallized Intelligence	Breath/depth of cultural knowledge, ability to communicate and reason verbally	73	3
Fluid Reasoning	Ability to reason, form concepts & solve problems, basic reasoning processes	97	42
Short Term Memory	Ability to learn and hold information, then use it within a few seconds	88	22
Long Term Retrieval	Ability to store information and retrieve it, associative storage and retrieval	85	16
Visual Processing	Ability to perceive, analyze, synthesize and think with visual patterns and store/recall	94	35
Processing Speed	Ability to perform automatic cognitive tasks under time pressure- attention and speed	97	42
Auditory Processing	Ability to analyze, synthesize & discriminate auditory information- not comprehension	103	58

Martin and Gabriel's math classroom. Martin's and Gabriel's fourth grade mathematics teacher, Mr. Guzman, organized his classroom with individual desks placed in rows of five seats and had a total of 18 students in his class. I observed Mr. Guzman's class on three separate occasions[3]. Mr. Guzman's instruction generally consisted of a

[3] My goal was to make observations of all the participating children in my study to obtain an idea of the type of delivered mathematics instruction. Because I received consent from Martin and Gabriel's teacher (not Julia's), thus I observed Martin's and Gabriel's engage in mathematics during Mr. Guzman's class.

presentation of content utilizing a document camera or white board accompanied by verbal questioning on the presented content. As students worked, Mr. Guzman would ask individual children questions about how they solved the problem and checked their final answers. In this classroom space, Martin sat in the back with a peer in the same table, different from the rows of desks, whereas Gabriel sat by himself in a corner table, close to Mr. Guzman's desk, away from the rest of his classmates.

Observations of Martin and Gabriel. Martin participated only when prompted by Mr. Guzman whereas Gabriel participated whether asked to or not. Gabriel at times became distracted and got his peers off task. Martin and Gabriel seemed to be following instructions when asked by Mr. Guzman, and at times would get distracted. Martin would eventually answer and write what Mr. Guzman asked him to write on his paper and most of the time answered his teacher's questions. Gabriel seemed to be engaged in the lesson, and be attentive to what his peers were saying and doing, and would find ways to get himself and his peers off task.

4.3 The Advancement of Children's Mathematical Agency

The following section illustrates each child's mathematical agency and participation as the sessions progressed. I present a description of certain teaching experiment sessions as they progressed and, embedded in them, the mathematical agency exhibited by specific children. Session descriptions are organized as specific instances where certain children exhibited mathematical agency throughout the study. I do not present all details of my actions as the instructor and the agency enacted by each child in each session. Rather, I include the instances that stood out to me as informative and pertinent to each child's mathematical agency development across the sessions presented in either the problem solving or discussion phase. In certain sessions, I embed descriptions of shared spaces among the children as they enacted varying forms of co-constructed agency through their participation during the problem solving and discussion phases. The section concludes with an overall descriptions of agency patterns exhibited by each child as shifts began to occur. Agency is presented as described in the analysis of the methods chapter (see Table 3.6 for agency definitions of limited, developing and enacting) describing mathematical agency as children's power to make sense and take ownership of their mathematical thinking. Two of the main differences between *limited and developing math agency* are acts of making sense and acts of taking ownership. Limited agency involves a child not trying to make sense of the mathematical strategies used whereas for developing agency a child attempts to make sense of the strategies used. Further, in limited agency a child will most likely not engage in taking ownership of the solution used to take an action to show his or her thinking to others, whereas for developing math agency, that child may attempt to take ownership of the thinking or strategy by explaining his or her thinking to others. The main difference between enacting mathematical agency and developing agency refers to a child taking initiatives to take ownership of their mathematical thinking by explaining, justifying, defending his or her thinking to others and sometimes explaining and contributing to other's math ideas.

4.3.1 Session 1: Initial mathematical agency.

In the beginning session, I decided to introduce a word problem that was relevant to my previous interactions with Julia, Martin and Gabriel in the pre-session, where I read the story of *Jasper's Creepy Carrots*. For the story problem, I presented a join-change-unknown problem to obtain initial mathematical conceptions of each child and to discover what strategies each child decided to utilize (e.g. original strategy or standard algorithm with no connections to the context of the word problem). This choice stemmed from my hypothesis that most curriculums in public schools only present join result unknown problems (e.g. 14 + 5=___) where students use standard algorithms. Yet, the problem type was a secondary decision point, as my primarily goal was to uncover how each child engaged in conversations with each other about Jasper's carrot problem (i.e., initial mathematical agency of each child). I hoped the children would have competing strategies that then would create disagreements and discussion.

Launch of the problem. As we began the session, I first introduced the problem (Table 4.3) in both Spanish and English to all three children. I launched the problem by reading it out loud and asking them what the problem was about. Julia remembered Jasper, from the previous day where we did introductions and read Jasper's book. I began with questions like "How many carrots does Jasper have?" and "What is the problem asking us to find?". At first, Martin and Gabriel were reluctant to respond, but with further prompting (i.e., "I noticed you wrote something [strategy] on your paper. Would you please explain it to me?"), the children discussed that Jasper had four carrots and that, after his friends gave him more, he now had 12 carrots.

Table 4.3: Session 1 problems.

Problem Type	Problem In English and Spanish
Join Change Unknown	Jasper has 4 carrots. His friends gave him some more carrots. Now Jasper has 12 carrots. How many carrots did Jasper's friends gave him?
	Jasper tiene 4 zanahorias. Sus amigos le dieron más zanahorias. Ahora Jasper tiene 12 zanahorias. ¿Cuántas zanahorias le dieron los amigos de Jasper?
Join Change Unknown Extension	Jasper has 8 carrots. His friends gave him some more carrots. Now Jasper has 20 carrots. How many carrots did Jasper's friends gave him?

After the launch, I asked them to think of a way to solve the problem in a way that made sense to them. I encouraged them to work together and suggested a choice to work in pairs or alone. As Martin, Gabriel and Julia began to solve the problem in the problem solving phase, Martin and Gabriel decided to work together, and Julia decided to work on her own. Martin and Gabriel began to solve the problem by using a standard algorithm of 12 plus 4 to get to 16 (Figure 4.4).

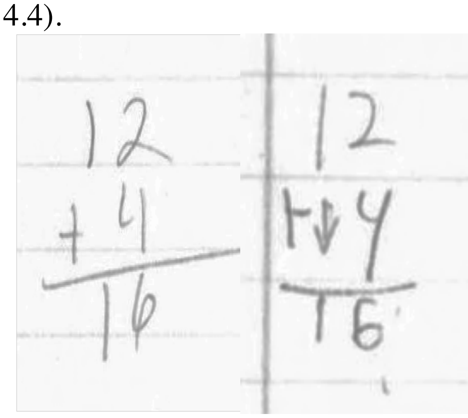

Figure 4.4 Gabriel (left) and Martin's (right) first solution to Jasper's problem

Martin and Gabriel's strategies. Martin and Gabriel worked together on this problem. They both used an addition of 12 and 4 together to make 16. To unveil Gabriel's mathematical thinking, I asked Gabriel to explain his strategy. Gabriel's response was a single answer of "16". When I asked for further explanation of his answer, he responded with "I don't know". Martin showed me his spiral with his work in it, pointing to his algorithm of 12 plus 4. I then asked, "How did you know to add 12 and 4 together? What in the problem told you this?" At this point, Martin began to read the problem out loud while Gabriel listened. As Gabriel listened he shouted out "oh its subtraction". When I asked, "Why do you think its subtraction?", Gabriel exclaimed, "I don't know", and shrugged his shoulders. Suddenly, I noticed Martin beginning to use his fingers to solve the problem. As I looked up to address him, Martin shouted, "It's eight". It appeared that Martin realized that Jasper's friends had given him some number of carrots and that now Jasper had 12. The following excerpt described my attempts to get him to explain his thinking about the 8 carrots:

> Martin: [Begins counting with his fingers and counts upward to 8] Oh he got… I know how much they gave him. They gave him 8 carrots [Uses his eight fingers and taps them on the table].
>
> Teacher: How did you get 8?
>
> Martin: Cuse, he had 12, Jasper has 12. I count… I added 8 to 4 and that gave me 12.
>
> Teacher: How did you get 8?
>
> Martin: By counting
>
> Teacher: How did you count? I saw you doing something with your fingers? [Teacher wiggles her fingers up in the air and smiles at Martin]

Martin: Yeah, I counted with fingers

Teacher: Can you show me what you did with your fingers? I am interested in [Teacher shakes her fingers once more].

Martin: [Smiles at Teacher] I was counting with my fingers one, two, three [Martin begins using his fingers and using his other hand to gesture his count, by covering each of his fingers as he counts] four, five, six, seven, eight. Then I added it to 4.

Teacher: You added it to 4?

Martin: Yeah.

Teacher: How did you add it to 4?

Martin: [Martin smiles and stays silent, he looks puzzled]

Teacher: I think I understand your strategy Martin and I really like your strategy. Do you want to explain it to Gabriel?

Martin: [Martin looks over to Gabriel, and Gabriel looks back at Martin].

Teacher: Maybe that will help you think about it. Some more… because I really like your thinking. [Teacher moves Gabriel's chair to face Martin]

Teacher: Gabriel you want to listen to what Martin says?

Gabriel: Yeah

Teacher: A ver, Martin explícale lo que me dijiste ahorita [Let's see Martin, explain what you just told me].

I positioned Martin as having a valid strategy where he had figured out that his answer is eight. This positioning influenced Martin to share his strategy even though he was unsure as to how he got eight. He was still struggling to explain his thinking and seemed uncomfortable explaining why he got eight. I had observed Martin starting at four and counting up to get to 12, were he showed eight fingers at the end, and when he proceeded to explain his thinking he already had eight fingers, so he had begun to count the eight and four more to get to 12. So, at this point I asked him to explain his thinking to Gabriel to support his reasoning. The following excerpt described Martin and Gabriel's interactions when explaining how Martin obtained an answer of eight.

Martin: So, in total it's how much his friends gave him. So, he had hmm, 12 so you count… because if you add… I put 4 and I got…8

Gabriel: How you got 8?

Martin: Oh, by counting with my fingers. Because I was, first, adding numbers to 4 [Gabriel looks confused] to finding out how to hmmm… hmm.

Gabriel: Oh, you were counting by four!

Martin: Yeah [Does not seem convinced]

Gabriel: Oh!

Martin: You see.

Gabriel: [Gabriel nods yes and Martin then follows with a nod too] So you have to add… [Gabriel looks over to Martin's paper] eight [Gabriel writes 4 plus 8 is 12].

Martin: [*Martin shouts*] I was counting by fours!

Interpretations of Martin and Gabriel's mathematical agency. At the beginning of the first episode, Martin and Gabriel had begun using algorithms and seemed to exhibit a limited understanding about the context of the problem. They both had similar strategies that were not making sense to either of them. Martin began to make sense of the mathematics when he was prompted by the teacher to re-read the Jasper's word problem. This allowed Martin to think about what Jasper had, and what Jasper's friends gave him. Also, inadvertently, Gabriel's comment about "it's subtraction" sparked Martin to rethink what the question was asking him to solve. Therefore, Martin had made sense of the mathematics context at hand due to the verbal contributions made by Gabriel. Yet, Martin had not completely made sense of his own strategy.

What potentially seemed to promote Martin's understanding of his own strategy (i.e., explaining his thinking to Gabriel) dissuaded Martin from thinking further about his own ideas and convinced him that his own strategy was counting by four, as 4, 8, 12, as Gabriel suggested. Essentially, Martin had taken up Gabriel's ideas as his own, believing that he was counting by two fours to get to 8, instead of starting with 4 and counting up to 12. Thus, in this instance Martin had yet to develop complete ownership of his own mathematical thinking, taking up his peer's ideas as his own. In part, I believe this happened because Martin was yet to make complete sense of his own mathematical strategy. Thus, I argue that Martin had begun to exhibit *developing mathematical agency***.** Martin was developing math agency because he made sense of his mathematical strategy but took little ownership of his own thinking.

Opposite to Martin's engagement and understanding of the problem, Gabriel was focused on getting one right answer, and therefore was attached to "I do not know" responses. Gabriel in this situation exhibited *limited mathematical agency*, not being able to fully understand and make sense of the problem and thus producing a lack of ownership in explaining his answer 16. Gabriel at some point made an attempt to make sense of the problem but was unsuccessful at fully developing his mathematical ideas, when he said "oh, it's subtraction" when I restated the word problem. Gabriel did not move his mathematical thinking further or questioned why he thought it was subtraction. In retrospect, I believe I could have attended to his response of "it's subtraction" and further pushed him to think as to why it was subtraction. He had thought it was the subtraction of

12 and four to get his final answer, but Gabriel never attempted to subtract these two numbers in his paper or try to somehow subtract using his fingers or the unifix cubes.

Julia's strategy. Julia had solved the problem using a buggy algorithm (see Figure 4.5 for her strategy) adding 12 and 4 together to obtain 52. When I approached Julia to ask how she had obtained 52, she explained that "I put twelve plus four" and "one plus four is five and two plus nothing is two". Julia had begun with a description of her algorithm in terms of numbers or cubes. Julia had added the one in the tens place of the number 12 and the four to obtain five in the tens place and the brought down the 2 (see Figure 4.5). To help Julia make a connection between what was going on with Jasper's carrots I asked her to identify what the 12 and four represented in the problem and I then asked Julia to show another way to solve the problem. Julia then began to use squares in her journal that represented the 12 and the 4, counted the 12 and 4 by ones and concluded that the answer was 16. When I asked her to explain her strategy, she described the numbers in the story problem in context as boxes and blocks instead of carrots. In the following excerpt, Julia explains her new strategy.

Julia: I put four blocks and twelve block… boxes and I got 16

Teacher: You got 16?

Julia: [Julia nods yes]

Teacher: Ok, so let's explain… Explain to me how you got… why did you put four here, what is the four in the problem? [Teacher points to the four rectangles Julia created on her paper]

Julia: Because it said right here, right here [Julia points to the number 4 in the word problem then circles the number 4 and 12] and I put 12 right here, so then I added them, to count them…

I made further attempts to ask her what the 12 and the four represent but Julia continued to respond with “I put four then 12 and I count the boxes… total… and I got 16”.

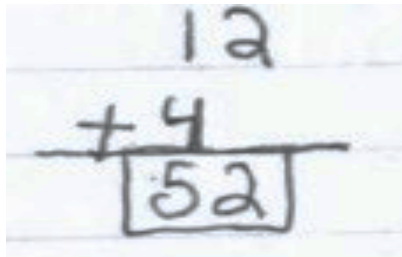

Figure 4.5 Julia’s first strategy in session one

Interpretations of Julia’s mathematical agency. This excerpt demonstrates how Julia thought about mathematics as doing operations (e.g. adding or subtracting) with numbers. She first continued by adding 1 and 4 together then 2 and nothing to obtain 52 as her final answer (see Figure 4.5). When asked to solve a different way, she used a counting strategy by adding the 4 and the 12 together and disregarded the context of the problem and what these numbers represented. Julia explained her strategy as adding numbers, for example she explained that she was adding 4 and 12 to get a total number of “boxes” to get a total amount of 16. Julia, in this instance, used procedures without making sense of the mathematical context. The four was yet to be “4 carrots” and the 12 as “12 carrots” to make a total of “16 carrots”: a context connection was yet to be visible to Julia. Thus, in this instance Julia was exhibiting *limited mathematical agency*.

4.3.2 Session 3: Martin refuses to share his ideas with Julia.

In the description below I explain Martin’s responses after asking him to please share his thinking with Julia during session three. I provide interpretations of Martin’s agency in that moment and how his agency developed from session one.

Launch of the problem. I began the third session by reading a book about toys in a train named *The Little Engine that Could*. I then introduced a task of a measurement division problem (see Table 4.4) that related to the story that I had read. During the launch, Julia decided she wanted to read the word problem in English to the group, and Gabriel decided to read it in Spanish. I related the problem context to the experiences of children's toys and how they placed them in bins after they were done playing with them. I asked them to go back to their seats and to solve the problem in any way they liked and to work together if they wished. Martin and Gabriel decided to work together, and Julia worked on her own.

Table 4.4: Session 3 problems.

Problem Type	Spanish	English
Measurement Division (with groups of 10)	Gabriel tiene 32 juguetes. Él pone 10 juguetes dentro de cada caja. ¿Cuántas cajas puede llenar Gabriel?	Gabriel has 32 toys. He puts 10 toys inside each bin. How many bins can he fill?

Children's strategies. Martin used a direct modeling strategy (see Figure 4.6) where he constructed three circles with 10 dots in each and had two leftovers. Gabriel solved the problem using a similar strategy to Martin's (see Figure 4.7), in his strategy he made a circle with 10 in each but he checked his answer by adding 10 and 10 to make 20 and the 20 and 10 more to make 30 plus the two leftovers. Julia in her strategy (see Figure 4.8) decided to add 32 and 10 using a standard algorithm to obtain an answer of 42.

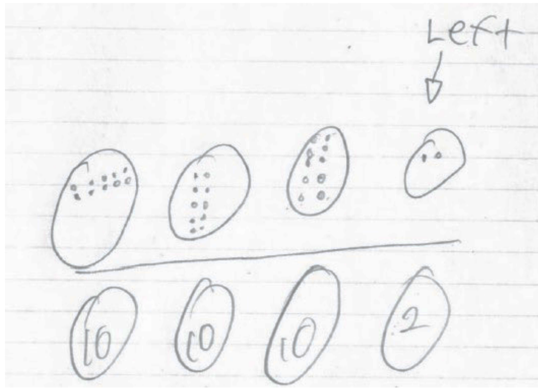

Figure 4.6 Martin's direct modeling strategy for session 3

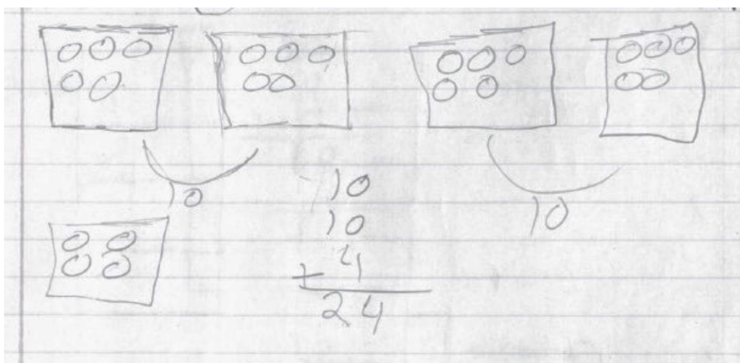

Figure 4.7 Gabriel's strategy for session 3

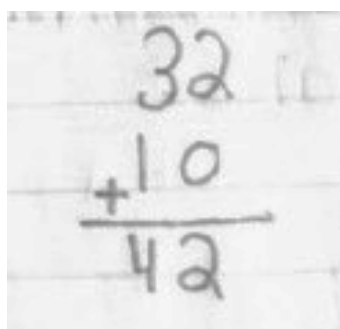

Figure 4.8 Julia's strategy for session 3

Instance of math agency exhibited by Martin. During the problem solving phase, Martin begun to explain his strategy to Gabriel and Gabriel seemed to agree with his thinking. I walked over to their group and asked Martin to further explain what he had done in his strategy which he then described as “I put 10 in each”, meaning that he had placed 10 toys in each of his circles representing a box, and “there are two leftovers”, meaning there were 2 toys leftover. On the opposite side of the table, I noticed that Julia had begun to add 32 and 10, asking, “Does he have 42 boxes?” as if she was unsure about her final answer. As I began to make my way to Julia’s side of the table. I noticed Julia had written 32 plus 10 equals 42 on her paper. She explained to me that there were 32 boxes and I pointed to the word problem.

Teacher: Gabriel has how many toys?

Julia: 32

Teacher: 32 toys. So, these are toys [Teacher points to the 32 in the word problem]. The 32 is the number of toys. Ok, and the 10 is how many toys are inside each box. So, what we want to find out is how many boxes he can fill.

Julia: Mmm.

Teacher: How many how many toys can he put inside each box? [Teacher points to the word problem] And you said… he puts how many?

Julia: 10 toys

Teacher: Mhm

Julia: So how many boxes can I fill?

I made the decision to ask Martin if he could help Julia explain his thinking to help her make sense of the problem. The following described what happens next.

Teacher: Martin, could you please explain your strategy to Julia?

Martin: [Looks at the teacher] Mmm.

Teacher: Maybe that will help her think about it, how she would want to solve the problem?

Martin: [Stays silent]

Julia: Wait I think I got the problem [Julia begins to grab some unifix cubes from the table] with blocks… I am going to use blocks…

Teacher: [Teacher leaves Julia to work, and goes to ask Gabriel about his strategy] Can I see your strategy Gabriel? [Gabriel shrugs his shoulders]

Teacher: [Teacher notices Julia grabs 32 cubes and 10 more so teacher decides to ask Martin once more] Martin, can you explain it to Julia what you did?

Martin: [Bites his lower lip and continues to be silent]

Teacher: Show her your strategy?

Martin: [Martin looks at Gabriel and smiles]

Teacher: [Teacher smiles and laughs] [Martin also smiles and smirks at Teacher] Porfavor? *[*Please?] I really like your strategy Martin, I really think it could help her think about how she wants to solve it.

Martin: [Smiles and squints eyes]

Teacher: A little bit… [Teacher makes gestures with hand indicating something small*] Poquitito. [*A little bit, laughs]

Martin: [Begins to shake head and continues to stay silent and smiles]

Interpretations of Martin's mathematical agency. Martin immediately solved the toy problem using a direct modeling strategy with circles and dots on his journal. He also explained his strategy to me and to Gabriel, but when I asked him to explain his strategy to Julia he refused. Despite my many attempts to position Martin as having expertise, in this situation it did not seem to promote the sharing of his mathematical strategy with Julia, therefore I would argue that Martin's agency was exhibited as resistance to engage in explaining his thinking with Julia. He refused to share his thinking with Julia, and thus inadvertently reduced Julia's opportunity to share her thinking with a peer and opportunity to make sense of the mathematical concepts attached to the context of the problem. Martin's decision to not share could have stemmed from Martin viewing Julia as limited in her mathematical understanding, or he could have been shy in explaining his thinking. Martin's agency was *limiting* in that he failed to fully take an action on his mathematical thinking with Julia, but *developing* in that he was able to make sense of his strategy.

Shifts in Mathematical Agency. At the beginning of session one, Martin had exhibited agency as *developing* because he had made sense of the Jasper word problem and shared his mathematical understanding with Gabriel and myself. In previous interactions during session one, I had not explicitly asked Martin to share his thinking with Julia and I wondered what would have been the responses when asked to share his thinking with her. Martin's agency had shifted from *developing* to *limited* and seemed to be highly dependent on who he was sharing his thinking with. In this instance, Martin's agency was shown as resistance. As I continued to develop and teach lessons, I wondered if this resistance to share with Julia would change in future lessons.

4.3.3 Session 4: Gabriel hides his counting strategy

In session four Gabriel struggled relating the context of the problem to his strategy. After several attempts on my part to change and relate the context of the problem to something that was interesting to him, Gabriel had begun to use a counting strategy, which he then hid. In this section I describe Gabriel's agency and his shift from session one.

Launch of problem. On the day of session four, I decided to introduce a multi-step problem due to my previous interactions with Martin, Gabriel and Julia. I anticipated that the problem would be a little challenging so I prepared alternatives for individual children, by changing the numbers or solving a simpler problem (e.g. Multi-step multiplication to multiplication problem, see Table 4.5). I began launching the problem by discussing who in their household had parents or grandparents that liked to make tamales. I followed by reading the problems in English and Spanish, since Julia preferred the problem read in English, and Gabriel in Spanish. Julia and Gabriel were the only children present during this session, Martin had left at the beginning of this session due to an appointment with the eye doctor. Julia only worked on the first problem, whereas Gabriel worked on all three problems.

Table 4.5: Session 4 problems.

Problem Type	Spanish	English
Multi-step multiplication problem (Base 10)	La abuelita de Gabriel tiene 9 paquetes de tamales. Cada paquete tiene 10 tamales en él. También tiene 6 tamales extras. ¿Cuántos tamales tiene la abuelita de Gabriel?	Gabriel's abuelita has 9 packages of tamales. Each package has 10 tamales in it. He also has 6 extra tamales. How many tamales does Gabriel's abuelita have?
2nd Multi-step multiplication problem (Base 10)	La abuelita de Gabriel tiene 2 paquetes de tamales. Cada paquete tiene 10 tamales en él. También tiene 6 tamales extras. ¿Cuántos tamales tiene la abuelita de Gabriel?	Gabriel's abuelita has 2 packages of tamales. Each package has 10 tamales in it. He also has 6 extra tamales. How many tamales does Gabriel's abuelita have?
3rd Multiplication Problem	Gabriel tiene 3 paquetes de 6 balones de soccer en cada paquete. ¿Cuantos balones de soccer tiene Gabriel?	Gabriel has 3 packages of 6 soccer balls in each package. How many soccer balls does Gabriel have?

Children's strategies. Julia solved the first problem 9 packages of tamales with 10 in each and six extra using two different strategies. She used a direct modeling strategy at first adding nine, 10 and six drawn squares together. After some teacher moves to get Julia to understand that packages and tamales were different items, her second strategy involved the use of skip counting by 10's up to 90, and then skip counting by 10's up to 60. Julia interpreted the six extra as packages and not tamales hence her strategy became an addition of 10 sixes, instead of 6 more ones. Gabriel solved the first problem using a standard algorithm, by first adding the nine and 10 together then adding 19 and six together to obtain 25. He added 19 and six together using the borrowing method adding nine and six to obtain 15 borrowing the one and carrying down the two.

Instance of mathematical agency exhibited by Gabriel. After noticing Gabriel's strategy for the first problem, I decided to give him a similar problem with simpler numbers, two packages, 10 tamales in each, and six extra. Gabriel at the time, seemed to be struggling to make sense of the context of the problem and continued to use his previous strategy, were he added all three numbers using a standard two-digit addition algorithm (see Figure 4.9).

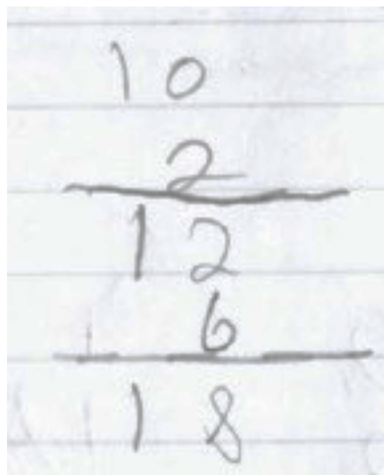

Figure 4.9 Gabriel's solution for the 2nd problem in session 4

I considered that Gabriel was not making a connection to the problem context because he was not interested in finding how many tamales there were, so I changed the problem to a simpler problem (see Table 4.5 for a description of the third problem), where I also strategically changed the context from tamales to soccer balls. Gabriel explained how his answer was nine because he had added six and three together. I asked him if there was a second different strategy he could use. As I turned my back to him, and began to attend to Julia's strategy, I noticed he began to use his fingers. Gabriel began counting with 6 fingers, then repeated his actions two more times, as if he was counting up to six, then counted up six and six more:

Gabriel: [Gabriel shouts] It is 18.

Teacher: I saw you using your fingers.

Gabriel: No.

Teacher: Its ok to use your fingers, I use my fingers all the time. Like how many brothers and sisters do I have, 1, 2, plus me that's 3. How did you use your fingers?

Gabriel: [Silent]

Teacher: Did you count 1, 2, 3, 4, 5, 6, [Pauses] 8, 9, 10…or were you doing…

Gabriel: [Shakes his head from side to side indicating a no] I was, I added 6 and 6 that's 12 and added another 6 and it was 18.

Teacher: Another six and it was 18?

Gabriel: [Gabriel nods yes]

Teacher: And did you do 6 in your head, or did you count 1, 2, 3, 4, 5, 6, and then you did 7, 8, 9, 10, 11, 12. Is that what you did?

Gabriel: [Nods yes]

Teacher: Then you added the last 6 to the 12

Gabriel: [Nods yes]

Interpretations of Gabriel's mathematical agency. Gabriel used a counting strategy to solve the soccer ball problem but only after I turned my back on him. He seemed to add three sets of six ones together, counting 1-2-3-4-5-6, 7-8-9-10-11-12, and 13-14-15-16-17-18 with his fingers, keeping track of the five fingers in one hand and 1 finger in the other each time. Although he clearly used his fingers to solve the problem, he denied it when I asked him. It was only when I stated that it was "Ok, to use your fingers" and further prompted by explaining a hypothesis of what he had done, only then did he begin to share what he had done. At first Gabriel shook his head indicating that was not what he had done but at the same time he exclaimed that he had added 6 and 6 to get 12 and then 6 more to make 18. Then I further elicited his thinking by asking him if he had counted in his head, or if he had counted with his fingers, to which he indicated he had counted with his fingers by ones. Gabriel in this instance enacted *developing agency*, where he made sense of the problem but failed to completely take ownership of his strategy. He denied having used a counting strategy. It was only when I provided reassurance that it was ok to use his fingers that he shared what he had done with his fingers. I inferred that he refused to share what he had done with his fingers because it was discouraged in his general math or special education classroom.

Shifts in Mathematical Agency. At the beginning of session one, Gabriel had exhibited agency as *limited* because he had failed to make sense of the Jasper word problem and refused to take an action on his mathematical understanding. In previous instances during session one, Gabriel explained his final answer but failed to share how he had obtained his solution. In session four, I began to attend to Gabriel's interests and payed close attention to his hidden strategies, and only then was I able to see Gabriel making sense of the math concepts. Gabriel's agency had shifted from *limited to developing* and was highly dependent on making the tasks meaningful to him. In this instance, Gabriel's agency was shown as *developing* because although he had made sense of his counting strategy he was reluctant in sharing his thinking and further taking complete ownership of his mathematical thinking. In future sessions, I made it a point to always keep an eye on Gabriel's hidden strategies exhibited during the problem solving phase, and in making the word problems relevant, and I wondered if this reluctance to share and take ownership of his strategies would change in future lessons.

4.3.4 Session 5: Gabriel and Martin defend their ideas

In session five, Gabriel interacted with and responded to Martin about his strategy during the problem-solving phase. These descriptions provide a rich example of how Gabriel's and Martin's co-constructed agency when explaining and making sense of the soccer ball task. I provide descriptions of their agency and how their agency shifted from previous sessions.

Table 4.6: Session 5 problems.

Problem Type	Spanish	English
Measurement Division problem	Messi tiene 24 balones de fútbol. Messi pone 5 balones de fútbol dentro de una bolsa. ¿Cuántas bolsas puede llenar?	Messi has 24 soccer balls. He puts 5 soccer balls inside a bag. How many bags can he fill?

Launch of the problem. On the day of session five, I decided to introduce a measurement division problem attached to a context that was of interest to Gabriel due to my interactions with him in the previous session (see Table 4.6). In session four, Gabriel did not make a connection to the context until I changed it to something that was interesting to him. At the beginning of this session I had introduced a story word problem with Ronaldino, a famous soccer player, but later changed it at the request of Gabriel expressing his dislike towards this player and emphasizing that Messi, another famous soccer player, was a better choice. The problem was presented in both English and Spanish and I gave them the option of working together in groups or individually. At first, I was worried that if I made the problem about soccer Julia would be disinterested and would not solve the problem, but I was wrong. Each child decided to solve the problem individually.

Children's strategies. Julia decided at first to add the numbers 24 and five together using unifix cubes to get a final answer. When I approach Julia, she explained how she added 11 and 11 to get 22 and two more to get 24 then five more. After helping clarify the problem's context, Julia decided to break apart her 24 cubes into four groups of four groups of five cubes and one of four cubes. Which she then decided to add another unifix cube to her group of four cubes making it a bag of five as well. She exclaimed that Messi has five bags with five soccer balls in each and drew five boxes representing the bags, and placed five circles in each representing the soccer balls (see Figure 4.10).

Gabriel solved problem by making five boxes with five circles in four of the boxes and four circles in the last box, where the boxes represented the bags and the circles represented the soccer balls. Gabriel also added five and five soccer balls together and made a grouping of 10 and 10 indicated with lines in his paper, which he then added as 10, 10, and 4 vertically to obtain 24 (see Figure 4.11). Martin had originally solved the problem thinking there were four soccer balls in each bag and had six circles with four dashes in each, which he then erased after discussing his strategy with Gabriel to five circles with five dashes (some of which he interchanged with small circles in his drawing) in 4 of the circles and 4 dashes in one of the circles (see Figure 4.12).

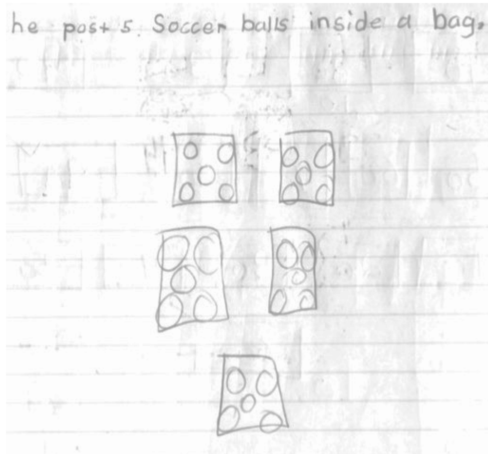

Figure 4.10 Julia's Strategy for Messi's Problem

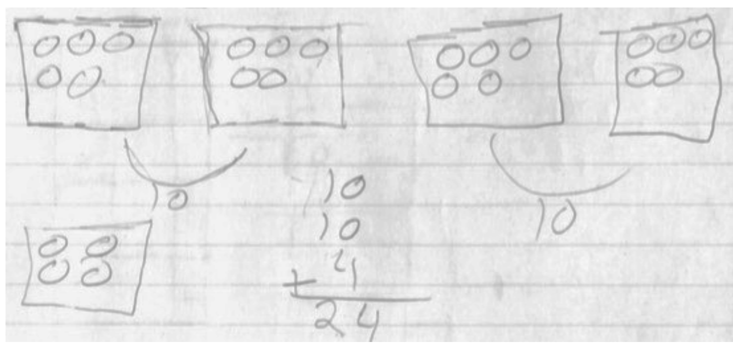

Figure 4.11 Gabriel's Strategy for Messi's problem

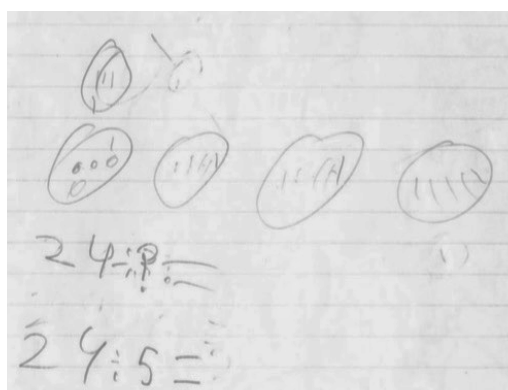

Figure 4.12 Martin's Strategy for Messi's Problem

Instance of math agency exhibited by Martin and Gabriel. During the problem solving of Martin and Gabriel exchanged mathematical ideas of their strategies used to solve the soccer ball problem. As I had predicted, changing the context of the problem to something that Gabriel could relate seemed to promote sense making. During the launch of the problem I noticed Gabriel immediately began to solve the problem before I finished discussing what Messi's problem was about to the group. I noticed Gabriel wanting to share his ideas almost immediately with Martin. The following transcript describes Martin and Gabriel's exchange of ideas on their strategies used.

[Martin and Gabriel are sitting on the floor. Martin works on his strategy and Gabriel looks over his shoulder. Martin solves by making six circles and four dashes in each to represent the soccer balls.]

Gabriel: Six?

Martin: [Turns and looks at Gabriel] Yeah, he has 6 bags

Gabriel: It is five [Gabriel shows Martin five fingers and references his strategy on his paper]

Martin: Yeah [Martin smiles at Gabriel]

Gabriel: Why did you put six? [Gabriel begins counting to himself the dashes on Martin's paper to verify it is 24] 23?

Martin: No, there is 24 right here [Martin assures Gabriel there are 24 dashes not 23, and smiles at him]

Gabriel: Anyway, you have to put five on each of them [Gabriel points to his circles and dots on his paper] not four

Martin: [Martin looks down at his paper which has 4 dashes in each circle then looks at the problem written on the board, pauses for a second, and begins erasing]

Five, ten, fifteen, twenty [Pauses, looks up to the ceiling as if thinking about something, erases a circle] Oh so there is going to be one leftover [Looks up at the word problem written on the board]

Gabriel: Que… Que? [What… What? Gabriel approaches Martin's paper once more]

Martin: There is going to be one leftover bro [Points to each of his circles and counts to show Gabriel] five, ten, fifteen, twenty, twenty-five… so we have…

Gabriel: [Gabriel interrupts Martin and takes Martin's spiral away from him and looks at his paper] How much is that? [Gabriel points to the dashes on Martin's paper]

Martin: Five

Gabriel: Five, five, five, five, and… aquí en esta? [Here in this one? Gabriel points to the bag with 4 dashes]

Martin: Four

Gabriel: En esta? [In this one? Gabriel grabs his pencil and points at it]

Martin: Cuatro

Gabriel: [Points at each of the dashes inside] Cuatro

Martin: Si, y luego la… [Yes, and then I added the…]

Gabriel: Y luego [And then] I added them… so its 24

Martin: [Erases some more of his previous work, and looks his spiral and places it down on the floor]

In this exchange of ideas between Martin and Gabriel I noticed several instances where Gabriel was trying to convince Martin that his strategy was correct. Gabriel initiated the conversation by questioning Martin's final answer of six bags. Martin seemed confident about his answer of six bags because he mistakenly thought the problem said four soccer balls in each bag. Gabriel's questioning did not seem to discourage Martin from keeping his answer, so Gabriel took it upon himself to explore the details of Martin's strategy. Gabriel began to count the dashes on Martin's paper to see if he in fact had 24 soccer balls. Martin assured Gabriel that he did in fact have 24 dashes on his paper. Then when Martin began justifying his own strategy Gabriel noticed Martin only had four dashes in each of his circles and not five. After noticing the fours inside the circles, Gabriel began to argue with Martin, stating that the problem said five (soccer balls) and not four as Martin had in his strategy. At this point, Martin was convinced that his strategy was in fact supposed to be five and not four soccer balls in each bag thus followed by some erasing and changing his strategy to five dashes, or soccer balls, in each.

Interpretation of Gabriel and Martin's mathematical agency. In this instance, both Martin and Gabriel were making sense of their own strategies and defending and arguing against their peer's mathematical ideas. Gabriel began questioning Martin's final answer but soon turned to attending to the details of Martin's strategy to then arguing against it. Martin was convinced his ideas were correct and had to be convinced otherwise. Thus, both made sense and took ownership of their mathematical ideas and strategies and therefore both were *enacting mathematical agency*.

Shifts in Mathematical Agency. During session one, Martin had exhibited agency as developing when engaging in sharing his thinking with Gabriel because he failed to capitalize on his thinking when explaining his counting strategy and taking Gabriel's interpretation of his strategy as his. In this session, Martin argued against Gabriel's ideas, and further justified his thinking stating that he did have 24 dashes or soccer balls in his strategy. Martin not only shared his thinking but justified his thinking with Gabriel thus shifting his agency from *developing* to *enacting*. In session three, Gabriel struggled to make sense of the problems and refused to share his counting strategy as he began to make sense of it. He hid his counting strategy from me and furthermore was hesitant in sharing his thinking with me when I asked him about it. In this session, due to my intentional attempts to make problems meaningful, Gabriel had immediately solved the problem in a way that made sense to him and further argued with Martin about his final solution. Gabriel was able to justify his solution and attend to the details of Martin's strategy to help him argue against it thus shifting his agency from *developing* to *enacting*. Yet, I wondered from these instances, if Martin would exhibit developing or enacting agency when sharing his thinking with Julia in future sessions. I also wondered if Martin, would begin to exhibit ownership of this ideas when explaining his thinking to me or to Julia. It was evident that Martin and Gabriel were extremely confident and comfortable sharing their thinking with each other, which was progress from earlier sessions.

4.3.5 Session 7: Julia is eager to share and Martin listens

In session seven, Julia demonstrated ownership and making sense of her thinking when she interacted with Martin. At the same time, Martin showed a change in participation when interacting and responding to Julia's mathematical reasoning. I provide descriptions of their agency and how their agency shifted from previous sessions.

Launch of the problem. The problem of the day was a multiplication problem that consisted of candy bars and bags (see Table 4.7). I made the decision to keep the context like the previous session since it promoted rich discussions among the children about their strategies. The problem was again read in English and Spanish and they had the option of working together or individually. Julia requested that I change the person's name of Maria to Jonasen, since she liked that name. During the launch, I tried relating the task to previous experiences with gummy bears or Jolly Rangers in bags being purchased at a grocery store. I asked if they had a favorite candy which they could imagine when solving the problem.

Table 4.7: Session 7 problems.

Problem Type	Spanish	English
Multiplication problem	Jonasen tiene 4 dulces en una bolsa, y él tiene 9 bolsas de dulces. ¿Cuántos dulces tiene Jonasen?	Jonasen has 4 candy bars in one bag. And he has 9 bags of candy bars. How many candy bars does Jonasen have?

Children's strategies. Martin solves the problem using two circles with four dashes in each and one more circle with one dash inside. Martin had interpreted the problem as Jonasen having nine total candy bars with four candy bars in each bag. So, he solved the problem by creating two groups of four and had one leftover. When he realized his mistake, he erased his work and reworked the problem to obtain nine bags with four candy bars in each (see Figure 4.13). Whereas Julia had solved the problem thinking there were nine bags and each bag had four candy bars (see Figure 4.14). Julia used a direct modeling strategy, by creating nine circles with four dashes inside each of her circles.

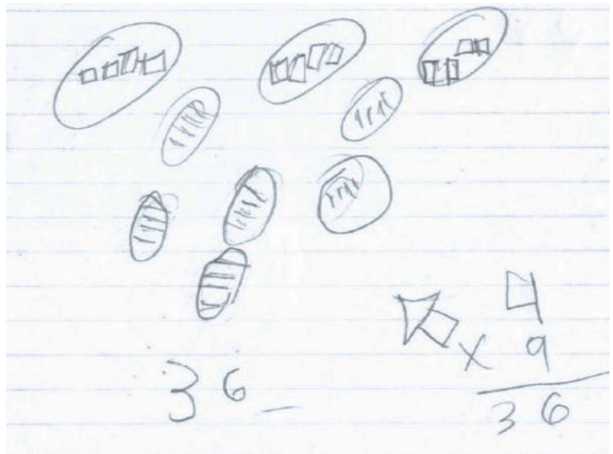

Figure 4.13 Martin's 2nd Strategy for Session 7

Figure 4.14 Julia's strategy for Session 7

Instance of mathematical agency exhibited by Julia and Martin. During the problem-solving phase, I noticed Martin had finished solving the problem quickly so I decided to investigate further into his thinking. While Martin began explaining his strategy, Julia began to speak out loud about what she had done with her strategy, and kept chiming into the discussion of Martin's strategy. At the time, Julia seemed eager to explain her thinking, so I followed up and proposed that they share their strategies with each other. The following instance is a discussion that occurred after Martin and Julia had finished solving the problem.

> Teacher: Ah, you guys have different strategies, do you guys want to share with each other what you guys did?
>
> Julia: You can go first Martin
>
> Teacher: Do you want to… Martin?
>
> Julia: I'd love to hear it
>
> Martin: [Martin pauses for about 5 seconds] I put 4 candy bars…
>
> Julia: You have 9?
>
> Martin: Hmm?
>
> Julia: You have 9 total? [Julia looks at Martin's strategy]
>
> Martin: [Martin looks at Julia and then at teacher]
>
> Teacher: Nine what?
>
> Julia: 9, 9 things [Julia points to Martin's strategy]
>
> Teacher: What are those 9 Martin?
>
> Martin: They are candy bars
>
> Julia: [Julia now changes the conversation to speak about her strategy] I put 9 bags and there are 4 candy bars in each one. [Julia begins to lower her voice to a whisper] and I got 36
>
> Martin: [Martin begins to write in his journal]
>
> Teacher: Oh, so Martin did something different…
>
> Julia: [Julia chimes in again, this time with a stronger tone of voice] Martin, what I did is I put 9 bags and I put 4 candy bars in each bag [Julia points to each of the circles she drew in her strategy representing the bags] and I got 36

Martin: [Martin continues to draw on his journal]

The conversation about Martin and Julia's strategies began with Julia expressing an interest in the details of Martin's strategy. Julia almost immediately asked Martin about his strategy, and attended to the 9 dashes in Martin's strategy, asking him if he had "9 total", meaning 9 total candy bars in the problem. Julia was intrigued because she had obtained a total of 36. This led to Julia taking over the conversation, where she shared how she had solved the problem using 9 groups of 4 and obtaining 36 candy bars. Julia proceeded to explain how she solved the problem, stating, "I got 36" in a much lower voice tone as she finished her sentence, almost as if she was unsure about her final answer. As the conversation continued, and I tried to refocus the conversation around Martin's strategy, Julia decided to repeat her explanation of her strategy but this time with a tone of confidence and almost proudness saying things like "Martin, what I did is I". I wanted to see if Martin had understood what Julia had described in her strategy. The following transcript illuminates what happened after I asked Martin to explain Julia's strategy.

Teacher: Did you hear what she said? [Teacher directs question to Martin]

Martin: Um hum

Teacher: What… what did she say about her strategy?

Martin: [Stays silent for 3 seconds] She had... she put 9 bags

Teacher: Um hum...

Martin: And put 4 in each one

Teacher: Mmm, what do you think about her strategy? Do you agree or disagree with her?

Martin: I agree

Teacher: You agree, why…why do you agree with her strategy?

Martin: [Martin begins shaking his head] uhh…

Julia: [Julia smiles at Martin] Actually, when I say I agree, I don't know why… I just agree I don't want to be mean to people.

Teacher: [Smiles at Julia] Aww, well it is ok if you disagree, it is ok to say no [Teacher turns and looks at Martin] you don't agree. Mm, its ok to say no [Teacher shakes head]

Martin: I don't know why

Teacher: You don't know why, you have no idea? Uhm, Julia do you want to explain why you put four in each… in each of the 9 bags?

In this moment, Martin could articulate the details of Julia's explanation of her strategy but he was confused about why Julia had solved the problem in that manner. Martin had interpreted the problem as Jonasen having 9 total candy bars, 4 candy bars in each bag, and wanting to find out how many bags Jonasen needed. He was unclear, and therefore at first stated he agreed, but then expressed he did not know why he agreed.

Interpretations of Martin and Julia's Mathematical agency. In this episode, Martin at first was reluctant, but soon began explaining his thinking to Julia, and when I asked him to explain what Julia had done for her strategy he could articulate what she had done in her strategy. Martin was clearly confused with what Julia had done for her strategy, but he felt he should say he agreed with her strategy even though he did not understand it. Thus, Martin in this episode exhibited *developing agency*. Martin made sense of his strategy but had difficulty explaining and arguing the details of his strategy.

On the other hand, Julia took over most of the conversation when I asked them to share their strategies. She was at first so eager to share her thinking that she would keep chiming into the conversation Martin and I were having about his strategy, to the point where I had to acknowledge her ideas. As Martin and Julia began sharing, Julia questioned Martin's strategy asking if he had a total of 9 candy bars and she had a total of 36. At first, she seemed as though she was unsure about her final answer, whispering she had obtained 36 candy bars, but then she kept insisting that her ideas were in fact valid. Julia in this episode *enacted mathematical agency*. Julia made sense of the candy task and took complete ownership of her reasoning for the use of her strategy.

Shifts in mathematical agency. Martin at one time would have not shared his strategy with Julia. For example, during session three he did not participate in explaining his strategy to Julia, but on this day, he decided he would share. This was evidence Martin was beginning to shift his agency not only to sharing his ideas with Gabriel but was also beginning to share his ideas with Julia. Martin's agency shifted from *limited* to *developing* when sharing his thinking with Julia. I noticed that this experience made an impact on Martin views about Julia's ideas as valid when he actively began to listen to her ideas about how she had obtained 36 candies.

During session one and previous sessions, Julia exhibited limited agency in that several of her solutions evidenced themselves as standard algorithms or direct modeling strategies without a relation to the context of the problem. Julia would often be excited to share her thinking with me or during group discussions, but her reasoning would often be absent or short of connections to the word problems. Julia would often share her thinking with me and the group during discussions but was rarely given an opportunity to share her thinking with peers, mostly because Martin and Gabriel preferred to work together. In this episode, Julia abruptly chimed into a conversation between Martin and me and insisted on sharing her thinking. She was eager to show how much she understood the problem and explained her reasoning for this problem. Julia had made a breakthrough in this instance, showing her ownership and sense making, thus shifting her agency from *limited* to *enacting*. This abrupt and sudden change in Julia's participation made me wonder if she was beginning to gain confidence in her mathematical ideas and if this would show up in future sessions. I was excited!

4.3.6 Session 10: Martin and Julia work together to solve a difficult problem

In session 10, Julia and Martin pushed through a hard problem during the problem solving phase. Martin and Julia began to contribute to each other's mathematical ideas in an effort to provide a solution to Martin's strategy. Below, I provide descriptions of their agency and how their agency shifted from previous sessions.

Launch of the problem. In this session, I decided to introduce more equal sharing problems. This time, I decide that sharing 7 objects with 4 people would be more appropriate since I noticed Julia and Gabriel using half for one of their strategies in the previous session. I wanted to capitalize on this prior knowledge and see if they could easily partition leftovers in multiple halves. Also, I wanted to promote different strategies among all children and therefore create more meaningful discussions around their mathematical thinking. Before the session, I prepared an extension problem of 10 cookies and 4 kids (see Table 4.8 for further descriptions of the problems posed), but I also gave them a new problem in the moment, 20 cookies and 7 kids, after a suggestion was made by both Martin and Julia wanting to solve an additional problem with 100 cookies. I took their suggestions, after which they quickly realized 100 was too much, so I decided in the moment to change the problem of 100 cookies to 20 cookies instead.

Table 4.8: Session 10 problems.

Problem Type	Spanish	English
Equal sharing problem	Ms. Rodríguez compro 7 galletas para compartir con 4 niños. Ella quiere compartir las galletas igualmente ¿Cuánta galleta recibe cada niño?	Ms. Rodriguez bought 7 cookies to share with 4 kids. She wants to share the cookies so that everyone gets the same amount. How much cookie can each child get?
Equal Sharing (1st Extension of the cookie problem)	Ms. Rodríguez compro 20 galletas para compartir con 7 niños. Ella quiere compartir las galletas igualmente ¿Cuánta galleta recibe cada niño?	Ms. Rodriguez bought 20 cookies to share with 7 kids. She wants to share the cookies so that everyone gets the same amount. How much cookie can each child get?
Equal Sharing (2nd Extension of the cookie problem)	Ms. Rodríguez compro 10 galletas para compartir con 4 niños. Ella quiere compartir las galletas igualmente ¿Cuánta galleta recibe cada niño?	Ms. Rodriguez bought 10 cookies to share with 4 kids. She wants to share the cookies so that everyone gets the same amount. How much cookie can each child get?

Children's strategies. Martin solved the first problem, four people share seven cookies, by first giving each kid one cookie and splitting the last three into four parts. Julia solved the first problem by giving a whole to each person, the splitting the three leftovers into halves each, and the last leftover into fourths. Gabriel solved the problem by first giving a whole to each kid and cutting the last three leftovers into four parts, as did Martin. Gabriel then changed his mind about his strategy, and gave a whole to each kid, and partitioned the three leftovers into half and then partitions two leftovers again in half. For the second problem, Gabriel decided to go back to class and Martin and Julia insisted on solving a new problem. The following paragraph details Martin and Julia's productive struggle with the first extension problem to the cookie problem.

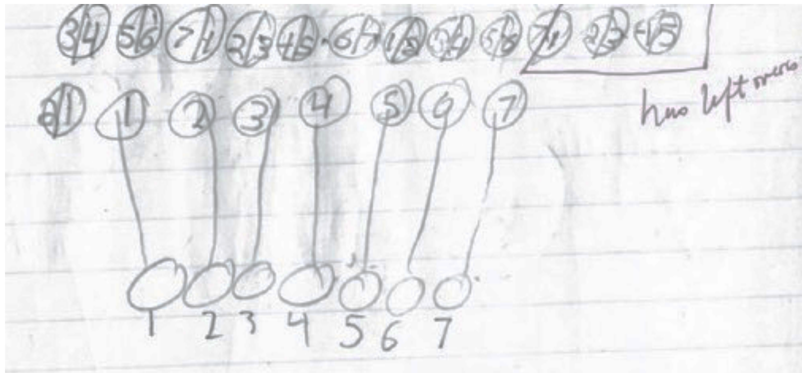

Figure 4.15 Julia's solution to the second problem posed in session 10

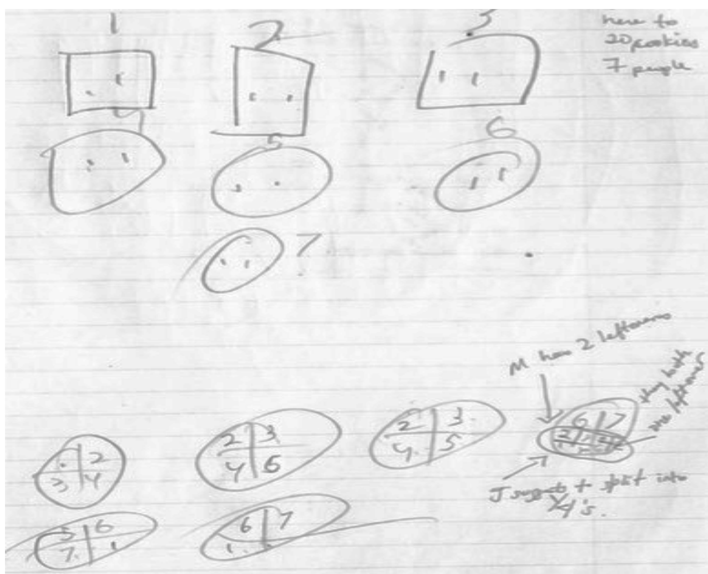

Figure 4.16 Martin's solution to the second problem posed in session 10

Instance of the mathematical agency exhibited by Martin and Julia. Martin and Julia were hard at work thinking and solving the 20 cookies and seven kids problem. They each had very different strategies. Julia gave each of the seven kids one whole cookie and proceeded to partition the remaining 13 leftovers into halves. She numbers her wholes and halves giving one whole and several uneven number of halves to each sharer (see Figure 4.15). Martin decided to give each of the seven kids two whole cookies and cuts the six leftovers into four pieces each. He began numbering his fourths one through seven. Martin accidently skipped over a fourth and forgot to give it a number so ends up with two fourths as leftovers (see Figure 4.16). During the discussion of his strategy I asked him what he could do with the two leftovers, to which he responded with "I do not know". Julia suggested cutting the last leftovers into four each. Martin used her idea and split the last two fourths into four pieces each. Martin numbered them and saw that he had one leftover. The following excerpt describes Julia's, Martin's, and my interactions while trying to support them in their struggle to solve the problem.

> Teacher: What do you do with that one leftover, how can you split it up? [Martin looks pensive]
>
> Julia: If I have one left? [Julia grabs pencil and begins 20 plus 7 in her journal]
>
> Teacher: If I have one left and I want to share it with seven people [Martin continues to be in deep thought]
>
> Julia: You can maybe… do… what is 20 plus 7? Or you can do… [Martin puts his hand on his cheek continuing to be in deep thought]
>
> Teacher: If you have one giant cookie and you want to share it with 7 people
>
> Julia: Let's see… [Julia directs question to Martin] what do you think?
>
> Martin: [Martin shakes his head] I don't know
>
> Teacher: Sometimes its ok to think about it, you don't have to give a right answer all the time… you can brainstorm… you can talk to each other…

Martin: Split them into eights?

Teacher: Split them into eights… so you are going to give each of those to each person [Teacher taps and points on the table]

Martin: There is going to be one leftover [Julia looks pensive]

Teacher: There is going to be one last leftover, what if it's a giant piece, how can we split it up?

Martin: Into sevens?

Julia: [Julia gets up and begins to draw a rectangle on the board and partitions it into seven pieces]

Interpretations of Julia's and Martin's agency. In this episode, Martin's agency was categorized as *developing mathematical agency*. But I would like to offer a description of the nuanced changes happening as the agency developed in the episode. Martin first solved the problem and had two leftovers, which he knew he had to do something with, but he did not know what. He stopped himself from pushing through the problem. But when Julia intervened and suggested to split the two leftovers into four parts, Martin took up her ideas and continued to push himself in solving the problem, thus continued to persevere. Julia's suggestions had influenced Martin to continue to make sense of the problem, thus influencing the agency Martin exhibited. Although I would like to categorize his agency as enacting, there were no opportunities to completely take ownership of this thinking in this episode. Julia had influenced Martin to continue to push through and their combined effort allowed them to make sense of the problem. If I would place a continuum for the category of *developing mathematical agency*, Martin would be placed at the far end, exhibiting almost close to *enacting mathematical agency*.

Julia exhibited *enacting mathematical agency* because she offered suggestions on how to partition the last item without any prompts from the teacher. In other words, she offered and took ownership of her mathematical ideas to contribute to a problem that still needed to be solved, while at the same time trying to make sense of those ideas.

Shifts in mathematical agency. Martin in session seven had begun to attend to Julia's mathematical ideas but in this session, he not only attended to her ideas but also utilized her ideas in his strategy. Martin had gone from beginning to see Julia's ideas as valid, to executing her ideas in his own problem solving, thus shifting his agency from listening Julia's ideas, *developing agency,* to engaging with Julia's ideas, to almost *enacting math agency*. In this instance, Martin had taken his and Julia's ideas as shared, thus co-constructing agency to understand a hard problem. Julia began to take ownership of her solutions by sharing with Martin and myself; in this session, she took ownership not only by sharing her thinking, but contributing her ideas to Martin's solution. Julia had both attended to Martin's strategy and contributed to help solve a difficult problem, thus exhibiting a stronger form of *enacting agency* in this episode.

4.3.7 Session 11: Gabriel takes an initiative

In session 11, Gabriel decided to take the initiative to solve a new problem in front of the group. Gabriel took a risk in solving an extension problem I created in the moment and decided to share his thinking with the group. Below, I provide interpretations of Gabriel's agency and shifts from previous sessions.

Launch of the problem. For this session, I decided to continue working with equal sharing problems since everyone really enjoyed these problems. I had a discussion with Martin about his abuelita selling candy in Mexico, so I wanted to make it relevant to his experiences. I was also curious to see how they would partition items with 7 people and if they would have very different strategies (see Table 4.9 for description of problems given). During the launch, I asked Martin to share his experiences with the group where he explained how she was always selling paletas de chile, a traditional Mexican Lolli pop that is covered with chili powder. All three children decided to work on their own.

Table 4.9: Session 11 Problems.

Problem Type	Spanish	English
Equal sharing problem	La abuelita de Martin tenia 11 dulces de caramelo para compartir con 7 niños. Ella quiere compartir los dulces de caramelo en partes iguales ¿Cuánto dulce recibe cada niño?	Martin's grandma had 11 candy bars to share with 7 kids. She wants to share the candy canes so that everyone gets the same amount. How much candy can each child get?
Extension problem	Compartir 3 dulces con 4 personas. ¿Cuanto dulce recibe cada persona?	Share 3 candy bars with 4 people. How much does each person get?

Children's strategies. Julia solved the abuelita problem of 11 candies shared among 7 people, by partitioning each item into 7 parts and writing the numerals 1-7 on each of them. Julia somewhere along the way lost track of the pieces and did not partition each bar into 7 but that was always her intent. Martin on the other hand, distributed 7 whole candy bars to each person and cut the 4 leftovers into fours. Martin realized he had two leftovers which he said he would give to other kids. Conversely, Gabriel solved the problem by giving a whole to each of the 7 kids and partitioned the two leftovers into half. Then after some discussion with me, Gabriel changed his strategy for the two leftovers by partitioning one into three parts and the second one into 7 parts.

Instance of the mathematical agency exhibited by Gabriel. In the last 10 minutes of the session, all children had shared their own strategies to the group. I was hoping to get different strategies to be able to discuss the comparison of each child's different ways of solving the problem. I did not orchestrate the conversation in the way I intended to; instead of continuing a conversation about their different strategies. I decided to pose a new problem of three bars and four people (see Table 4.9 for a description of the extension problem). I was not sure who would volunteer to solve or provide ideas, but I was curious to see what would happen if I engaged all three children in a new problem at the same time. To my surprise, Gabriel volunteered. The following excerpt describes Gabriel's initiative to solve the problem.

Teacher: If I have 3 candy bars [Teacher draws three rectangles on the board] and I have 4 people [Teacher draws four smile faces].

Gabriel: Naah that is easy!

Teacher: How would you share those candy bars? Martin?

Gabriel: [Gabriel gets up from his seat and approaches the board and grabs the marker from teacher] Like this look… [Gabriel begins partitioning each of the three bars in half. Then he tries to make four pieces in each bar but forgets to partition one of the bars into four pieces, does three instead.]

Teacher: Martin are you paying attention?

Martin: Yup

Julia: Are you doing the other problem?

Martin: Into fours

Teacher: Ah… are you guys… watching what Gabriel is doing?

Martin: He is putting them in three?? Four??

Teacher: [Gabriel begins to distribute the fourths to each of the smile faces] Where you trying to split that into four? [Teacher points to the bar that is partitioned into 3 parts]

Gabriel: I don't know how to do it. [Gabriel puts down the marker and sits down]

Teacher: But you are doing it right…

Gabriel at this moment seemed defeated and unwilling to continue solving the problem. I had inadvertently questioned his strategy by asking "Where you trying to split that into four?" which had an unfortunate consequence of stopping Gabriel from continuing to solve the problem. I was a little discouraged with my questioning, so I began to ask the group to describe what Gabriel had done. I wanted to let Gabriel know that I still valued his ideas and that I wanted to share them with the group. The following transcript describes my attempt to position Gabriel as an expert and Gabriel's response to this positioning.

Teacher: What did Gabriel do? [Teacher directs question to the group]

Gabriel: I split them up into four

Teacher: And then he did… [Gabriel and Julia talk but it is inaudible]

Julia: Oh, he said he is going to do another one. [Julia was referring to Gabriel going to the board again to redo the problem]

Teacher: Oh OK [Teacher Laughs]

Gabriel: [Gabriel gets up grabs a marker and begins erasing his previous work and redoing 3 bars with 4 unequal partitioned parts in each]

Martin: What is he doing? [Martin is wondering what Gabriel is doing]

Teacher: He is splitting up his candy bars. [Teacher responds to Martin] How is he splitting up his candy bars? [Teacher directs the question to the group]

Julia: Into fours

Martin: Into…

Teacher: Into fours. What is he doing now? [Gabriel is distributing the pieces to each person by placing line to each piece and to each person]

Julia: You can just put up 1-2-3-4 … 1-2-3-4… 1-2-3-4.

Teacher: Yeah you write 1-2-3-4, 1-2-3-4, and those can be the people, right? [Martin and Julia nod yes].

After positioning Gabriel. as competent and as having valuable ideas. Gabriel was inclined to erase his previous strategy and redo his strategy again. Gabriel, knew his plan was to create four parts in each of this candy bars all along, but had unsuccessfully done so in his previous drawing, so he then wanted to show the group what his goal was.

Interpretation of Gabriel's agency. Gabriel had successfully solved the problem and took it upon himself to the be the person to go to the board and share his ideas. When I asked Martin to explain what he would do, Gabriel took the initiative and showed the group how he would solve this problem. Furthermore, when he expressed that he felt he did not "know how to do it", I believe this was due to his realization he had made a mistake for one of his candy bars, and maybe thought I was being critical of his ideas. After my positioning moves, he took it upon himself to re-do his previous strategy to accommodate his original idea of cutting each candy bar into four parts, thus Gabriel had *enacted mathematical agency*. Gabriel had made sense of the problem and taken ownership of his partitioning strategy. It is important to point out that this was the first time Gabriel had taken the initiative to solve a problem in front of the group without previously consulting with peers or with me beforehand.

Shifts in mathematical agency. Gabriel, in previous sessions, had exhibited little agency when he refused to explain his solution in session one and hid his counting strategy in session four. He slowly began to develop ownership of his ideas when sharing and justifying his thinking with Martin in session five. It was not until this instance that he decided to act, in form of initiatives and risks in explaining and describing his thinking when solving a new problem. He not only described his thinking but he began to take the lead by initiating ideas within the group. Gabriel had gone from refusing to any action of his thinking to taking risks in sharing his thinking with others.

4.3.8 Session 12: Final Mathematical Agency Exhibited

In the section below I begin by explaining the story problems given and the strategies Julia, Gabriel and Martin decided to use in the final session. Second, I describe an instance of an initiated final discussion between all three children about their different strategies. Finally, I provide interpretations of all children's agency due to their interactions and suggestions when comparing each other's strategies.

Launch of the problem. For the final session, I wanted to give a problem where children could solve using several strategies (see Table 4.10). I also wanted to encourage children to share their ideas with each other, so I decided I wanted to make their strategies visible on the white board, as we had done in the previous session, to encourage children to attend to each other's strategies. I had been unsuccessful in attending to a comparison of strategies in the previous session, and I wanted to make it a goal for today's session. I noticed from previous sessions that making their strategies visible on the white board seemed to prompt more discussion and participation between children.

Table 4.10: Session 12 problems.

Problem Type	Spanish	English
Equal sharing problem	La mama de Janie trajo 4 brownies para compartir con 6 niños. Ella quiere compartir los brownies en partes iguales ¿Cuánto brownie recibe cada niño?	Janie's mom brought 4 brownies to share with 6 kids. She wants to share the brownies so that everyone gets the same amount. How much brownie can each child get?

Children's strategies. Martin had solved the brownie problem by partitioning each of his 4 bars into 6 parts each, distributing 4 parts to each of the 6 people. Julia had solved the problem by partitioning 3 of her bars into halves and giving one to each of the 6 people and then the last bar into 12 parts and giving two of those parts to each of the 6 people. Finally, Gabriel had partitioned each of his 4 bars into 3 parts each, distributing 2 parts to each of the 6 people. I was extremely happy to see very different and unique ways of solving the same problem.

Instance of agency exhibited by all three children. During the discussion of the children's strategies, I encouraged them to re-draw their strategies on the board. I first asked Julia to go up and draw her strategy, which she agreed to. Soon after Julia began drawing, Gabriel and Martin volunteered to write their strategies as well. I was not expecting them to go up at the same time, but they were too excited to share with the group so I let them go to the board (See Figure 4.17 for the details of their strategies). As it turned out, it was the perfect way to showcase all three strategies and to get them to compare their strategies.

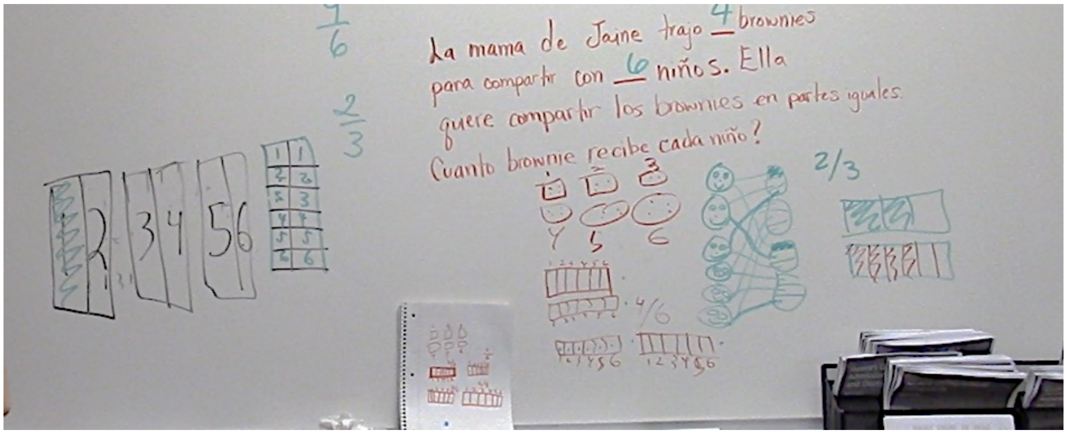

Figure 4.17 All Children's Strategies for session 12

During the discussion, I began to explain Martin's strategy when Gabriel chimed in to explain his strategy. I was attempting to get Julia to explain why Martin had decided his share was four out of six when Gabriel very briefly whispered "mine is 2 out of 3". The following excerpt describes the conversation Gabriel, Martin and I began to engage in.

Teacher: [Gabriel whispers mine is 2 out of 3] What did you say? [Teacher points at Gabriel]

Gabriel: I think mine is 2 out of 3. I don't know.

Teacher: Why do you think its minus 2 out of 3 [Teacher misinterprets and writes -2/3 on the board]

Gabriel: I said 2 out of 3 not minus

Martin: Yeah why do you think it's a minus?

Teacher: Oooh! That is yours [Teacher points to Gabriel's strategy on the board] So each person gets 2 out of 3 for you [Teacher points at Gabriel's bars partitioned bars]

Martin: Oh yeah!

Teacher: Each person gets 4 out of 6… How do you know [Teacher points to Martin] it's 2 out of 3?

Martin: Cuse… watch [Martin approaches the white board] I don't know [Smiles, laughs and turns away]

Gabriel: Cuse…

Teacher: Gabriel?

Gabriel: Cuse… I cut the brownies into 3 pieces and I give each of them two [Teacher shades in two parts for one person on Gabriel's strategy]

Martin: Oh yeah, he gives [Martin approaches the white board one more time and begins to point at Gabriel's strategy] one to this one [Martin points to the first shaded part in the first bar and one of Gabriel's people] and then one to this one and then… [Martin continues to point at each of the parts given to each of the people] …

Gabriel: And the brownies I cut them in three

Martin: [Smiles and returns to his chair] Wow!

Teacher: Ah! So, he cut his brownies into three, right? [Teacher draws a bar and cuts it into three parts] so I think each person gets two [Teacher shades two parts of the 3] and so that is why 2 out of 3 like you said [Teacher points to Gabriel]

Gabriel: Yeah

At this point, Gabriel had explained how his strategy was different but similar to Martin's strategy. Martin had explained how he obtained 4 parts out of 6 total of one brownie and now Gabriel was explaining how he had cut his brownies into three parts and each person got two parts, hence his "two out of three".

As the conversation continued, I explained to the group how Martin had cut his brownies into 6 parts and gave each person 4 of those parts, Martin chimed in and began to explain Julia's strategy. The following excerpt describes the discussion that arouse around Julia's strategy.

Martin: [Martin had interrupted the teacher] So, then this one is going to be [Martin gets up from his chair and points at Julia's strategy on the board. The teacher turns around and looks at Julia's strategy with Martin] two out of two… [Martin points at two of the halves Julia had in one of the bars labeled 1 and 2.]

Gabriel: Whaaat?

Teacher: Two out of two

Martin: Yeah, this one is two?

Julia: No!

Martin: Well I don't know

Teacher: Why do you think it's no? [Teacher turns and asks Julia]

Gabriel: So, each of them gets 2?

Julia: I split them in halves and I gave one to each people [Julia grabs the marker from the teacher and draws a circle representing a person and draws a line from one of the halves to the circle] and two to the other people [Julia draws two lines point to two of the 12 pieces in her last bar to the same circle]

Teacher: Ahh!

Martin: Ah.. mmm.. [Places a finger on his mouth]

Julia: And to the other people [Julia continues giving halves and two parts of 12 to new people (circles)]

Martin: Yeah, …

In this scenario, Martin had volunteered to explain how Julia had shared her brownies with each of the people, thinking there were two parts out of two, not realizing that Julia had cut the whole brownies into different sizes. Julia immediately exclaimed "NO!" as to indicate that Martin had interpreted what she had done in her strategy incorrectly, and followed with an explanation. Julia then began to draw the sharers, in her drawing they were circles, to represent how much each person would get.

Interpretation of all children's agency. All children had *enacted mathematical agency* in the discussion of their strategies. Gabriel had offered his own explanation, "mine is two out of three" of his final share, based on what he heard Martin say "it's four out of six". He was eager to explain, despite my misinterpretation of what he said as "minus" instead of "mine is". Not only had Gabriel taken ownership of his ideas, but he also made a connection between the details of Martin strategy and his. Before I could compare Julia's strategy with Martin and Gabriel's strategy, Martin volunteered to explain the differences between their strategies and Julia's. Martin had not only understood his own strategy and Gabriel's but also offered his own interpretations about Julia's strategy without it being prompted. Although it seemed at first that Julia was not engaged in the conversation about Martin and Gabriel's strategies, she began to defend her mathematical strategy when Martin had incorrectly suggested her strategy was "two out of two". Furthermore, Julia had understood that we were discussing each person's share because as she defended her strategy and began to draw the share for one person.

4.4 Limiting factors that Influenced Agency

During the teaching experiment, there were two predominate limiting factors that influenced the degree of agency exhibited by all three children. These two limitations were the space in which the sessions were held and unexpected visitors making their presence during the lessons. I will describe these below.

Location of sessions. The duration of the study was held in two different resource rooms**.** One, the conference room, had a large table with eight chairs surrounding it. It was supplied with a large white board and several materials like markers and erasers. This room was small but it had enough space to sit in a small area in front of the board separate from the table. Sessions 1, 3, 5, 8, 9, 11, and 12 were conducted in this room. Overall the feel of the room was appropriate and had enough space for children to spread out and work in groups or individually. Sessions were fairly organized in terms of the structure. The launch of the story problem could be conducted on the carpet near the white board, the problem-solving phase at the table, and the final discussion of children's strategies could be conducted at the carpet near the white board.

In contrast, the other room, a small section of the counselor's office, had a small student table that could fit about four student chairs total. This room was a lot smaller and did not have a large white board. The white board was about 3 feet in length by 5 feet wide. The space had an air unit installed on a window and it was usually turned on. The room was also filled with lots of little toys that the counselor had for her students. Sessions 2, 4, 6, 7, and 10 were conducted in this room. Overall the room was tiny and made it hard for children to work and concentrate.

These sessions were ill-defined and less structured than those in the conference room, and it was difficult to separate out the problem-solving phase and discussion phase for each of the sessions. Often, the sessions were blended during the exploration and discussion phases.

The spaces largely impacted the agency enacted by Gabriel. Gabriel would often be distracted with the toys in the counselor's room. Sometimes Gabriel would be working on a strategy and he would grab a toy he noticed on the corner of his eye and begin playing with it. Normally this would not be an issue, but at times it became too distracting. For instance, in session seven, his playing with kinetic sand became extremely distracting that completely deterred him from continuing to engage in conversations with his peers about his strategy.

Unexpected Visitors. Another limiting factor affecting the agency of the children was unexpected visitors into the sessions. I had two different instances of unexpected visitors throughout the duration of the study. One was due to the principal needing the conference room to hold a parent meeting. I had to move all three children to the other room and stop the session in the middle of the launch of a story problem. This was not a big deal, but there was a second visitor that made it extremely hard for Gabriel to participate in the discussion phase of session eight. Gabriel had excitedly shared his thinking about his strategy with Martin and myself during the exploration of their strategies, but when a visitor came in and asked to sit in the session, Gabriel became disengaged so much that he refused to share his strategy with the group, thus affecting his agency. This scenario is evidence of the importance to the children's mathematical agency of the relational dynamic we had established in the learning environment, which was disrupted by the appearance of an authority figure from the school.

4.5 Summary: Each Child's Evolution of Agency

Julia, Martin and Gabriel all exhibited limited, developing and enacting agency at one point in the twelve sessions. All had positive shifts in their agency from limited to enacting, meaning all children seemed to be gaining mathematical agency as the sessions progressed. Still, there were differences in the way that each of the children exhibited limited and developing mathematical agency. This section will portray how agency changed for each child in this study.

Julia's Mathematical Agency. Julia began the sessions one through four by exhibiting mostly limited mathematical agency with a few exceptions where she made sense of her peer's strategies and began to understand her own strategies (see Figure 4.18 for Julia's exhibited agency). Julia initially began using algorithms without making connections to the context and therefore failing to make sense of her strategies. As the sessions progressed, Julia began to abandon buggy algorithms and began using more robust strategies, like direct modeling, to solve word problems. It was not until session seven that it became apparent it was her goal to understand her own strategy and that of her peers. Julia was eager to share with her peers that she had finally understood her own strategies and was happy to explain those mathematical ideas with others. For example, in session seven, she completely took over the conversation when Martin was still trying to explain his strategy during discussion. As Julia's mathematical agency continued to develop, and she began to take ownership of her ideas, she moved from attending to the details of her own strategies to those of her peers. For instance, it was in sessions 9 through 11, where Julia began to offer suggestions to help her peers, Martin and Gabriel, solve their mathematical word problems. Julia had left behind the standard algorithms without connections to the context of word problems and began to adapt to new ways of thinking using her ideas and her suggestions as evidence for her strategies.

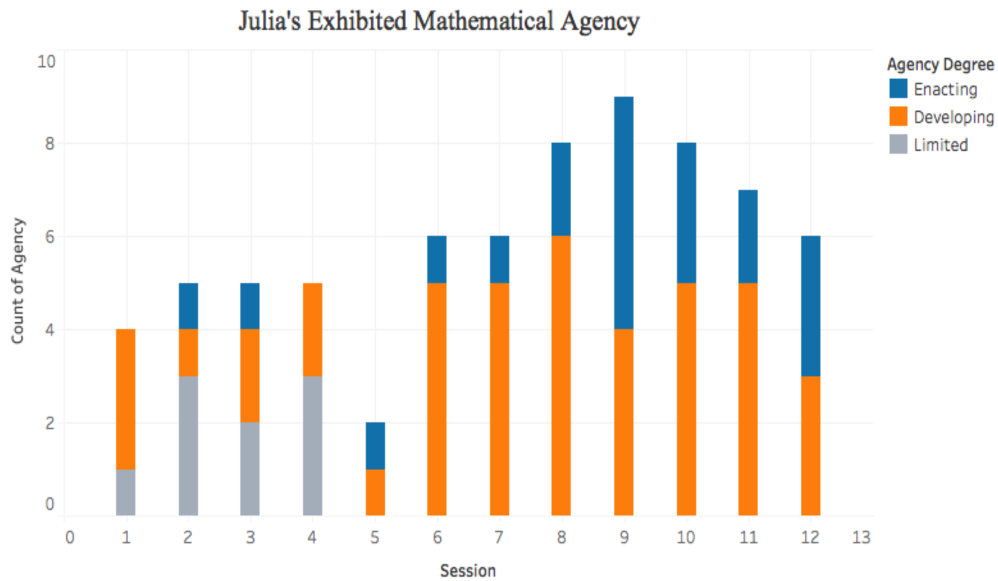

Figure 4.18 Julia's exhibited mathematical agency for session 1-12

Martin's Mathematical Agency. As opposed to Julia's beginning limited agency, Martin's limited agency presented itself in a different form. Unlike Julia, Martin immediately solved story problems in a way that made sense to him, abandoning standard algorithms without connections to the context in the first session. Martin always tried to make sense of his mathematical strategies in the given context, but it was with participating and sharing where his agency was needing. In sessions 3, 5 and 6, Martin either refuses to collaborate and share his thinking with a peer or refuses to share his mathematical strategy during discussion. Martin at several points, in sessions three and six, refused to work with Julia in helping her understand his thinking about the strategy he used. Martin also was reluctant and refused to share his thinking about a strategy Gabriel and he had worked on during the discussion of session 5, for which he delegated the explanation to Gabriel. It is during session 7 where Martin begins to exhibit changes in his agency (see Figure 4.19 for details of Martin's exhibited agency). Martin hesitantly began to share his thinking with

Julia about what he had done to solve a multiplication problem about nine bags and four candy bars. Where Martin clearly makes a significant change in his agency from developing to enacting is when he decides to work with Julia on a difficult problem in session 10. Martin takes up Julia's ideas as valid when they are trying to figure out how to split one leftover among seven people. Soon after this session, Martin begins to share his mathematical ideas with her and attend to the details of Julia's strategies, in sessions 11 and 12. Martin shares details to the group about how Julia partitioned her items in session 12 and describes how you could interpret one person's share in Julia's strategy. Martin had shifted in his participation from being hesitant and sometimes refusing to share his mathematical ideas to collaborating and taking up other's ideas to make sense of his own ideas.

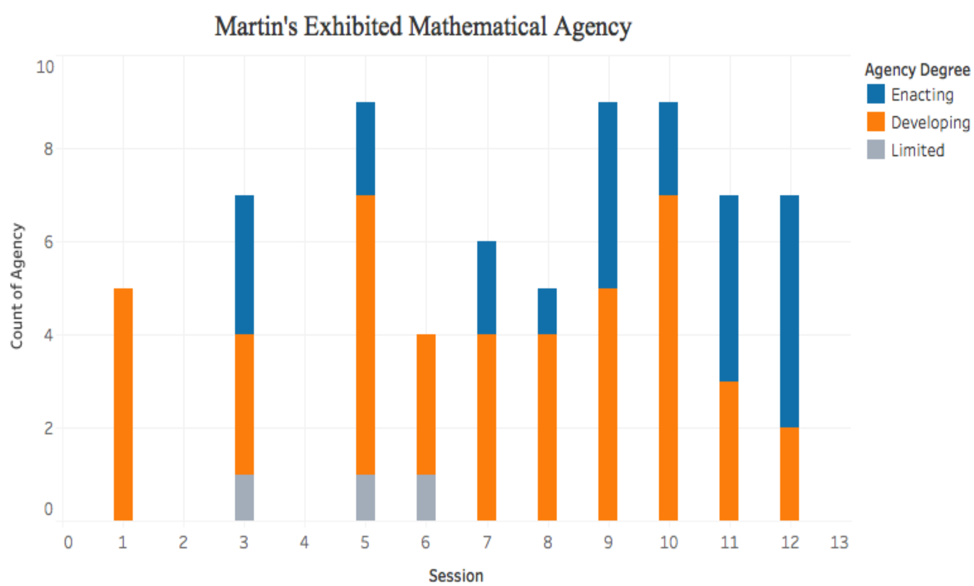

Figure 4.19 Martin's exhibited mathematical agency for session 1-12

Gabriel's Mathematical Agency. Like Julia, Gabriel began using standard algorithms without connections to the context to solve word problems. Initially in sessions 1 and 2 he would explain his final answer but refuse to share his reasoning behind the final answer or explain his thinking to the group. Unlike Julia and Martin, Gabriel seemed to be affected by his environment. At times, he would exhibit limited agency due to his distractions with gadgets available in the room. For example, in session 10 Gabriel was extremely distracted with a toy he found inside the room where the sessions were being held. He completely abandoned his mathematical strategy and became interested in playing with this toy. Despite the many distractions surrounding his environment, Gabriel, in session 5, began to engage in conversations with Martin about their strategies, to the point where he was attending to the details of Martin's strategy to argue and defend his reasoning for his correct answer. Defending his reasoning became part of his goal in the end and further taking risks in solving new problems on the board for the first time in front of his peers, as he had done in session 11. Gabriel overall gained a sense of pride in his ideas and his methods for solving problems where he no longer relied on me to explain his strategy to the group. Gabriel's pathway towards exhibiting higher degrees of agency began to show in sessions 9, 11, and 12 (see Figure 4.20 for Gabriel's exhibited agency). He began to take risks in sharing his ideas about his peers' strategies or in posing mathematical arguments.

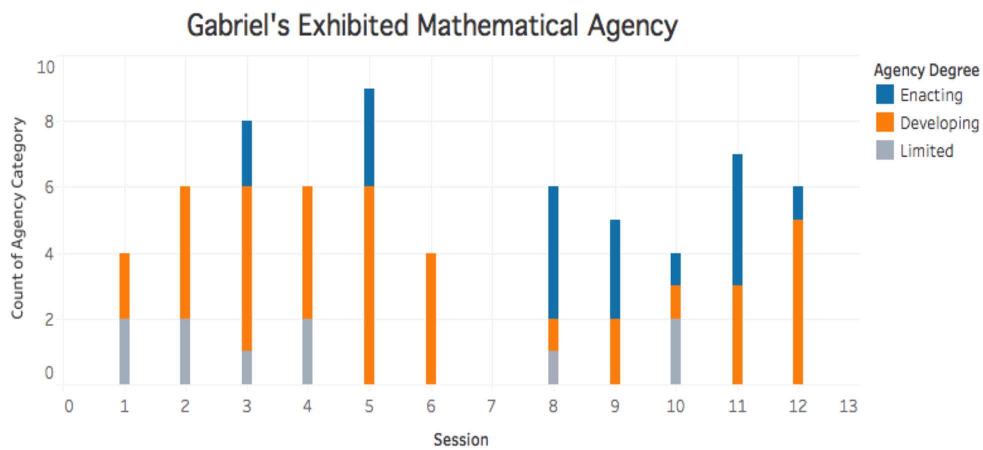

Figure 4.20 Gabriel's exhibited mathematical agency for session 1-12

Chapter 5. Discussion and Conclusion

In this study, I aimed to understand how Latino/a children with identified labels exhibit mathematical agency when participating in mathematical problem-solving discussions. More specifically, I sought to understand: How do Latino/a emerging bilingual children with identified learning disability labels and/or difficulties develop mathematical agency as mathematical learners through their participation in mathematical practices? In this chapter I will first discuss the themes that emerged in answering the research question. Next, I will discuss some of the implications of this study and directions for future teaching and research aimed at promoting the agency of Latina/o emerging bilinguals with identified labels within mathematics education. Finally, I will describe some of the limitations of this study and recommendations for future research.

5.1 Discussion of Mathematical Agency in relation to existing literature

In chapter two, I discussed the literature that informed the research question and directed the design for this study. In this section, I describe each of the factors that emerged as part of the embedded case studies as they affirm, contradict and contribute to the literature. Further, I describe additional literature that was helpful in the interpretation of the findings.

Contrary to prior research that supports explicit direct instruction (e.g. Gersten et al., 2008; 2015), the three Latino/a emerging bilingual children with identified struggling or LD labels in this study were exposed to instruction that builds on children's thinking (Carpenter et al, 2015). The instruction in this study focused on problem solving discussions with embedded mathematical practices that support teaching mathematics for understanding (Jacobs & Empson, 2016; NCTM, 2000). The mathematical practices explicitly attended to helping struggling Latino/a children make sense of the mathematics (e.g. Empson, 2003; Hunt and Empson, 2015), allowed children to use their native language (Gutierrez, 2008; Moschkovich, 1999, 2002; Planas & Civil, 2013), and provided opportunities to position children as competent learners and doers of mathematics (Greeno, 2003; Gresalfi, et al., 2009; Turner, 2003; Turner et al., 2013; Herbel-Eisenmann, Wagner, Johnson, Figueras, 2015; Wagner, Herbel-Eisenmann, 2009). This study demonstrated the potential of Martin, Julia and Gabriel, three lively emerging bilinguals with identified LD or labeled as struggling with mathematics, evidenced by a positive shift towards increased mathematical agency. Specifically, the three participating children exhibited increasing agency that emerged as three distinct themes: (a) use of algorithms to original strategies, (b) displaying little ownership of mathematical ideas to taking initiatives to share and justify mathematical ideas, and (c) moving from maintaining neutrality of peer's mathematical thinking to contributing to their peer's mathematical thinking. The three themes describe shifts in how children's agency changed from *limited* towards enactment of *mathematical agency*. These shifts occurred both in terms of making sense of the mathematics and taking ownership of each child's participation during the teaching experiment.

Shifts from limited to high sense making of solutions to word problems. In the beginning sessions, all three children evidence a use of standard algorithms with little sense making to solve addition and multiplication word problems. For example, Julia and Gabriel followed procedures to solve word problems using an algorithm without attending to what the problem was about. They enacted *limited agency* by primarily being interested in performing the correct operations and obtaining a correct answer instead of trying to make a connection between the algorithms used and context interpretation of what the problem was about. Before the initiation of the study, I hypothesized that the three children would show evidence of what Boaler and Greeno (2000) identify as surrendering agency by assimilating into the classroom culture of follow procedures at the beginning of the sessions, and that it would take time for children to re-construct their definitions of what mathematics learning is. This hypothesis stemmed from knowledge about research on children identified with LD being exposed to instruction in special education and math classrooms that is mostly focused on memorization and explicit delivery of procedures (Fuchs et al., 2005; Gersten et al., 2015), where children are rarely given opportunities to create their own mathematical strategies. This hypothesis was confirmed, when all children in sessions one and two used standard algorithms with limited understanding of what the problem was about or with little attempt to understand how the algorithms were making sense as solutions to the word problems given.

It was not until the middle of the teaching experiment sessions when Julia began to favor direct modeling strategies that attached to the context of the problem, Gabriel began to use unique ways (e.g. invented strategies) of solving word problems, and Martin began to make connections between his strategies and symbolic representations. This positive shift favors what mathematics education researchers (e.g. Baroody & Dowker, 2003; Fennema et al., 1996) see as important steps to fostering children's deep understanding of the mathematics. It is important to point out that as these positive shifts began to occur in sense making so did the children's ownership of their mathematical thinking. In several instances, it was observed as both.

Shifts of little ownership of mathematical ideas to taking initiatives to share and justify mathematical ideas. Standard algorithm strategies, with little connection to the context of the word problems, would usually be followed by no or little ownership of children's mathematical thinking at the beginning of the sessions. Little or no ownership would present itself as a form of resistance to participate in small group discussions or during one-to-one interactions. For instance, Gabriel and Martin would refuse or hesitate to explain their strategies during group discussions and sometimes would hide strategies. In session four, Gabriel hid his counting strategy when he explained that he had added three sixes to get a final answer of 18 to solve a multiplication problem. In session three, Martin demonstrated little and no ownership by being reluctant to explaining his counting strategy during discussions and refusing to share his thinking with Julia. The initial demonstration of little or no ownership of the participating children could be attributed to their comfortableness with working in a new environment, individual identities and personalities, and/or it could potentially be attributed to learned behaviors from their own special education and math classrooms, whose focus is on learned procedures.

In research about Latina/o children on agency as resistance, researchers like Cavell explain that such behaviors can be attributed to instruction focused on memorizing procedures because it fails to connect children's prior experiences in and outside of school (2011). Also, researchers like Lambert (2015) explain that the agency exhibited as resistance could be attributed to instruction that focuses solely on teaching to a standardized exam and could potentially be the main contributor to the construction of a mathematics disability (2015). Agency as resistance was evident in early sessions with both Gabriel and Martin, but as they continued to be engaged in instruction that focused on using children's prior knowledge (Van de Walle, Karp, & Bay-Williams, 2010) and positioned them as competent learners (e.g. Cobb, Wood, & Yackel, 1993; Gonzalez & Dejarnette, 2015; Greeno, 2009; Gresalfi & Martin, et al., 2009), resistant behaviors began to shift.

Shifts in the participating children's agency began to occur as they engaged in the construction and sharing of their own strategies. For example, in session five Martin and Gabriel each took it upon themselves to create their own direct modeling strategies. They began to take initiatives to share and defend their thinking with each other. Gabriel expressed his disagreement with Martin's strategy of making groups six groups of four soccer balls, stating that there should be four groups. Martin actively listened to Gabriel's strategy explanations but continued to believe that his strategy was correct and thus defended it. Gabriel seemed to be unsuccessful at convincing Martin, until he began to attend to the details of Martin's strategy (e.g. how many soccer balls he had drawn and how many where present in each group) and provided enough evidence to defend his strategy. Over time, Gabriel seemed to gain confidence in sharing his thinking by engaging in justifications his mathematical strategies to whole group discussions. And Martin began to engage in justifications of his' and his peers' strategies to whole group and one to one interactions. Martin and Gabriel's ownership of mathematical ideas shifted from refusing to share their thinking to justifying their mathematical thinking to peers, the teacher, and small group discussions. Martin and Gabriel's participation in justification of a solution is a significant finding because it is considered one of the main contributors to children building mathematical understanding (Chapin, O'Conner, and Anderson, 2009; Hiebert et al., 1997; Common Core Standards). These findings offer a reason as to why these practices (e.g. Stein et al., 2008; Turner et al., 2013) are fundamental and helpful in building mathematical understanding and, at the same time, build agency. More specifically, these practices are potentially essential to building the agency of emerging bilingual children with identified struggling or LD labels.

Shift in maintaining neutrality of peer's mathematical thinking to contributing to their peer's mathematical thinking. Findings in this study also confirm some of the results found in Webb et al.'s (2014) study of six teachers and 111 children on problem solving discussions. In their study, they found a positive correlation between the level of engagement of children in whole and small group discussions and mathematics achievement. They delineated a range of engagement as producing low to high levels of engagement as it correlates to achievement. For instance, some of the levels included children (1) participating in discussions by explaining one's own thinking, categorized as a low level of engagement, (2) attending to the details of a peer's mathematical thinking, a medium level of engagement, (3) contributing details to another peer's mathematical ideas, high level engagement, and (4) working together to co-construct a strategy, high level engagement (2014). As in their study, I found all four categories as evidence of engagement in this study. Specifically, this study confirms Webb et al.'s four categories, and I argue that they are also indicators of each participating child's mathematical agency. Each child's level of engagement with peer's ideas became more predominant as the sessions progressed. Discussion of the first two categories were explained in the section above (e.g. justifying children's thinking), this section focuses on the last two: (1) adding details to peer's ideas and (2) working together to co-construct a strategy.

At the initiation of the sessions, all three children expressed agreement with their peer's mathematical strategies but were usually followed with responses such as "I do not know why I agree, I just do" or "I am agreeing because I do not want to be mean". Children usually took their peer's shared strategies as true, and rarely questioned the validity of them or expanded upon them. I observed all three children engaging in what Mercer (1996) refers to as *cumulative talk,* where children do not make it a goal to challenge each other's thinking or did not see the need to explain their agreement.

Shifts began to occur towards the end of the sessions, when all three children in some way engaged and contributed ideas to a peer's mathematical strategy. For instance, Julia in session 10, offered a suggestion on how to partition the last item for Martin's strategy, Martin in session 12, shared his interpretation of Julia's partitioning strategy for quantifying a person's share as "two out of two", and Gabriel offered a comparison between Martin's quantified share and his own quantified share as "mine is two out of three". Children contributing to other's mathematical strategies became the new social norm within the group. Children began to actively engage with their peer's strategies and attend to interpretations and extensions of their peer's mathematical thinking. This is a powerful finding in describing how the agency increased for each child in this small group intervention because it showed the level of ownership that was attributed to their own thinking and the thinking of their peers. Furthermore, this ownership was unprompted on my part, thus expanding upon the empowerment children exhibited in this environment.

Another related and powerful finding in increased agency occurred in session seven, when Julia and Martin worked together to co-construct a solution to a strategy for a difficult equal sharing problem. Not only were they engaged in attending to each other's thinking but they were also interested in finding a solution for partitioning a leftover amount that made sense to both. Thus, agency exhibited by both children as *enacting mathematical agency* at its highest level, with children sharing ideas, listening to other's ideas, and taking each other's ideas seriously not needing my approval. The exhibited ownership worked to help Martin and Julia feel empowered to continue pressing each other to understand despite encountering a difficult word problem.

5.2 Implications of the Study

In this study, I explicitly attended to mathematical practices that position Latino/a children identified as struggling and LD labels as mathematical doers and experts (Empson, 2003; Turner et al., 2013; NCTM 2000). The practices of helping children make sense of the problem, clarifying, and extending children's thinking (e.g. Jacobs & Empson, 2016) were used to leverage each participating child in this study to help in promoting children's own mathematical thinking. The practices of positioning Latina/o children as experts and evaluators (e.g. Turner et al., 2013) were used to help increase their participation by gaining ownership of their mathematical thinking.

This small environment produced powerful interactions between Martin, Julia and Gabriel, that produced high levels of mathematical agency. The mathematical practices leveraged in this study potentially allowed all three children to evidence positive increases in their mathematical agency. Specifically, the teacher moves and types of tasks used in this study were in part, a major contributor to the agency exhibited by the participating children. In fact, without the teaching moves or the types of tasks I believe it would have been more difficult for children alone to enact high degrees of mathematical agency. In fact, the learning environment created with the mathematical practices was fragile, as noted in the findings, when an unexpected visitor interrupted the type of agency that Gabriel could exhibit. Further research on a small group of children with identified struggling or LD labels should be conducted to confirm or support the findings of this study.

Small group vs individual intervention. The teaching experiment in this study was conducted in a small group, as opposed to a one-to-one environment, mainly because I wanted children to engage in peer interactions that would allow me to position children as experts and doers of mathematics among others. Because my positionality in the study, acting as the teacher, could have altered and contributed to the limited agency exhibited by all children, a one-to-one environment would have been more restrictive. My reasoning for a small group intervention was guided by other researchers like Amit and Fried (2005) and Gresalfi and colleagues (2009) who explain that the authority of the teacher has high influence on the agency enacted by children. They discuss that the role of the teacher is usually perceived by the child as the person who holds all or most of the power, therefore limiting the agency a child could enact. In several instances, the participating children in the teaching experiment had an opportunity to collaborate with peers, providing several opportunities for them to share, contribute, or challenge each other's thinking. Hence, these group discussions provided opportunities for children to enact high levels of mathematical agency by seeing themselves as powerful agents and sometimes producing a co-construction of agency. But a least restrictive environment, or one-to-one environment with similar teaching moves, types of tasks and norms could be established that engenders the same type of agency as evidenced in these small group environments. Further studies should be conducted with children identified with learning disabilities in a one-to-one teaching experiment with similar mathematical practices and instruction to identify how they exhibit mathematical agency, as it is an open question as to whether one-to-one environments can produce the same kind of agency found in this study. In these studies, research questions could be investigated that seek to find degrees of agency exhibited and if the duration of the sessions yields a similar or higher level of agency.

Advocating for instruction focused on problem solving and mathematical practices in studies of special education. Research on teaching and learning of children with identified labels that documents explicit direct mathematics instruction found resistance as the main form of exhibited agency (Lambert, 2015). Similarly, the two

Latina/o children with identified labels, Ana and Luis, in Lambert's study exhibited behaviors resembling the initial findings of my study with Martin and Gabriel. Unlike my study's instructional intervention, she found evidence of persistent behaviors to exclude and reduce a child's agency in the math classroom when the focus was on memorization of procedures to the extent that this type of instruction was contributing to deficit views of the child as a learner. The findings of this study, along with Lambert's (2015), provide a basis for continued research on small group interventions that focus instruction on problem solving with embedded mathematical practices. The majority of research in special education research supports explicit, directed approaches (Woodward, 2004). Yet, if the aim of research and practice is to increase the agency of emerging bilingual children with identified LD labels, then I suggest that special education and mathematics research integrate mathematical practices into instruction to promote the use of children's prior knowledge in future studies. In this way, I challenge researchers to conduct similar studies and document not only children's agency but also children's mathematical learning.

Giving choice in problem solving sessions. In this study, I chose to provide all three children with choice in how they were to solve word problems. I provided children with a choice to read and discuss word problems in English and/or Spanish during the sessions. I also provided children with a choice to interact and solve problems individually or in groups.

All three emerging bilingual children chose to speak mostly in English, but at times would shift from English to Spanish. Children at the beginning of the sessions chose to read and speak in English, but towards the end of the sessions they felt empowered to speak in Spanish when they did not understand or could not express their thinking in English. For example, Gabriel would often prefer the word problem read to him in Spanish, and would switch between English and Spanish in the same sentences. Julia had a hard time reading in Spanish, but sometimes attempted to read problems in Spanish. I noticed that when I restated the word problem in Spanish during the problem-solving phase this helped her understand the context of the problem. Previous research on use of native language as a resource (Garcia, 2009; Garcia & Kleifgen, 2010; Garcia & Wei, 2014) to help children learn new content in large part influenced my decision to provide a choice of language use during the sessions. Being an emerging bilingual person myself also greatly influenced this decision and it sometimes helped me translate English to Spanish to better understand a new concept. Emerging bilingual children use code-switching between English and Spanish to communicate ideas and that this form often occurs outside their classrooms on an everyday basis. I wanted to provide these opportunities to help empower children to communicate ideas in whatever language they felt more comfortable. Language choice became a large part of the study, and I wonder if, had I analyzed the data further, if agency was positively correlated with the use of both languages.

A second choice provided to the children in this study was the option to solve problems in groups or individually in order to communicate ideas and solve problems. This decision would allow me to see if group or individual arrangements would change over time. I wanted to find out whether children found it easy or preferred to interact with other students whom they had not interacted with when initiating the sessions. This would allow me to see how authorship shifted among the group and individual interactions. Muller, Yankelewitz, and Maher (2012) explain that the form of authority children experience can influence the agency exhibited by a child. They posit that certain children can choose to work with or be influenced by an individual in a group, that a child could view this person as the sole expert. Hence, a child may choose to only work with certain peers over others, or look for acknowledgement from that person. In the findings of my study I observed that Julia tended to work alone and Martin and Gabriel tended to work together at the initiation of the sessions. This preference could have been due to many factors (e.g., gender preferences, friendships formed outside this environment, or viewing peers as less/more competent). I suspected it had to do with viewing each other as less/more competent, because Julia was in third grade and Martin and Gabriel were in fourth grade. Towards the end of the sessions, the group dynamic changed: Gabriel was inclined to working with Julia, and Martin began to listen and take on Julia's mathematical ideas as valid. Thus, Martin and Gabriel abandoned their preference to always work together and allowed Julia into their group conversations. I am left wondering what could have happened if sessions would have continued- would all three children prefer to work alone or in alternating gender groups?

Teaching implications. Math and special education teachers of emerging bilingual children with struggling and identified LD labels should pay close attention to the type of instruction they currently use and how that gives or reduces a child's agency. I invite teachers to learn more about their students' dispositions and interests because these might help create more meaningful mathematics discussions. I urge teachers to allow emerging bilingual children to utilize their native language to speak, discuss and share their mathematical thinking, for these opportunities might allow children to learn a mathematical concept that seemed out of reach before. Finally, I invite teachers to use the tools (e.g. teaching moves, types of tasks, and norms) explained in this study to provide opportunities for children to exhibit ownership of their mathematical thinking.

I implore teachers to consider the time and careful attention it takes to learn how to negotiate mathematical practices that expand children's choices of language use, strategy use, and participation during problem solving and mathematical discussions as they consider their own instructional practice. Children's rich agency will not be immediate when engaging in this type of instruction by simply engaging in more group work during math instruction. Instead, I assert that this way of teaching is by no means easy and will probably take time. In other words, I know that it is difficult to teach in this manner yet teachers *can* teach this way. As Harry and Klinger (2007) expressed, "Rather than devoting extensive resources to finding out whether [children] "have" disabilities, we should devote those resources to assessing" children's *abilities* (p.16).

5.3 LIMITATIONS OF THE STUDY AND FUTURE RESEARCH

The focus of this study was on the agency exhibited by three emerging bilinguals with identified difficulties and LD labels. The findings of this study as it pertained to the small group environment suggest that these children are capable of exhibiting mathematical agency that promotes the learning of math concepts with understanding, but it does not explain whether a relationship exists between the mathematics agency exhibited and the learning of mathematics concepts (e.g. base ten and fractions). Thus, there are questions that are yet to be answered: (1) Does a relationship exist between math agency and the learning of mathematics concepts? (2) If it does, is this relationship positively correlated?

A similar and related question that could be analyzed with the data already collected is: Is there positive correlations between the complexity of a strategy explained by a child and the enactment of mathematical agency? For example, at the beginning of the session, Julia utilized procedures without connections to the context of base ten problems and towards the end of the sessions Julia created sophisticated strategies to solve equal sharing problems. Similarly, Martin's explanations of his strategies became more complex as the sessions progressed. He began to include direct modeling strategies of adding seven groups of four with a multiplication number sentence (e.g. 7 X 4). Research on the complexity of the strategies shared by individual child and the agency enacted need to be explored further.

There are further limitations of this analysis that I would like to explore, such as (a) Are there specific mathematical teaching moves (e.g. clarifying, extending, assigning competence) that promote or hinder mathematical agency? This question is particularly interesting to me because, positioning children as experts or evaluators of ideas seemed to most often promote actions that encouraged children to explain their thinking with others and gain confidence in accepting their solutions as valid. Also, at particular times, I noticed certain children rejecting or devaluing their ideas based on a question I had posed about the specifics of their strategy. For instance, when Gabriel took an initiative to solve a newly posed problem on the board in front of the group, I asked him, if he had intended to cut one of the bars into four parts instead of three (which is what he had on the board). I assumed his goal was to cut all his bars into four pieces each, and therefore took it upon myself to help clarify this. In the moment, Gabriel interpreted my question as judgment that his strategy was incorrect or somewhat flawed. This seemed to prompt a reaction of distress and, inadvertently, promoted him to give up by siting back down. I realized that the question I posed had made Gabriel feel as though his ideas were incorrect, so I began to ask the group about his strategy, and positioning him as an expert followed by praise on my part (e.g. "this is a great strategy"). This anecdote is one that I can clearly remember as hindering the agency that Gabriel exhibited, but I am certain there are others that I am not accounting for that are present in the data. Because I did not explicitly look for more evidence of this happening I am wondering if there were more moments were the agency of a child was decreased.

Future studies. While I continued to read about social justice and math equity research I came about a study (Turner, Drake, McDuffie, Aguirre, Bartell, & Foote, 2012) that included children's multiple mathematics knowledge bases. Turner and colleagues documented ways in which teacher participants used children's multiple mathematics knowledge bases. I found these practices intriguing in that they not only utilized word problems that included familiar situations based on children's interests and prior experiences outside the classroom, but also authentic home and community situations (e.g. approximating money a mom will spend at the grocery store or how to maximize use of quarters at a laundromat). Such word problems were authentic to students' prior knowledge and experiences and included community and family experiences. In a future study, I would like to use a different approach, that integrates children's math thinking, cultural, home and community based knowledge and documents the type of agency exhibited by Latino/a emerging bilinguals' with identified labels. I wonder if the mathematical agency exhibited would be similar or richer.

5.4 Conclusion and Recommendations

The primary reason I chose to study marginalized groups of students was due to some of the injustices I observed in general and in special education classrooms with children with identified LDs. First, I noticed a large representation of emerging bilinguals within this population in high poverty schools (Artiles, 2005; Sullivan, 2011, 2013) and deemed it important to document these students' stories. Secondly, I noticed a large area of research recommendations for instruction in special education advocating for direct explicit instruction (Brosvic, Dihoff, Epstein, & Cook, 2006; Freeman & Crawford 2008; Freeman 2012; Gersten et al., 2008; 2015; Orosco, 2014) with an aim in improving math performance over learning. These observations influenced my decision to focus on the *agency* exhibited by emerging bilingual children struggling or identified with LD, as increases in agency are directly related to students' learning (Boaler & Greeno, 2000; Stein, Remillard, & Smith, 2007). I sought to investigate the agency exhibited by these children when instruction was focused on making sense of the math and choices and confidence were given to use their native language, work in groups or individually, and to use their own strategies, and opportunities were available to engage in math discussions.

The findings of this study reveal that these three children were "able" to make sense of word problems, took ownership and engage in conversations about their mathematical thinking, and were empowered to co-construct mathematical thinking with peers. Yet, even after documenting Martin, Julia, and Gabriel's agency stories, I am left with more questions than answers regarding classroom interactions in spaces aimed at promoting the learning of mathematics and positioning students as competent. I wonder whether a small group environment was more appropriate than one-to-one environments. I wonder whether their agency was also positively correlated with the learning of base ten and fraction concepts. I wonder if the choice to use their native language increased the agency enacted. I wonder, if I would have integrated more authentic problems related to their home and community, would the agency have been richer?

After the conclusion of the study, I have engaged in conversations with colleagues whose aims align with providing underrepresented children with choice and opportunities in math education research and teaching, and I have learned that there is much to accomplish in this area. For instance, there is a high need for research to document what instructional practices are being utilized with children with identified labels and how these practices could be affecting their agency in the mathematics classroom. There is also a need for teachers of special education programs to be aware of other types of math instruction that does promote mathematics learning, not just performance, with the aim at shifting the agency of a child toward ownership and empowerment.

Appendix A

Table Description of coded Teacher Moves

Categories of Teacher Moves	Description
Categories of Teacher practices during the Launch of the problem	
Teacher helps children make sense of the story problem	Teacher introduces the story problem by using a familiar story context. The teacher usually engages in an informal conversation about the story describing the word problem. The teacher generally asks children to explain what the story problem is about and usually helps children understand what the story problem is asking them to solve. During the children's explanation of what the story problem is about, the teacher could ask specific children to describe the details that they know about the story problem (e.g. How many cookies does Juan have? "How many cookies go inside each box?") Finally, the teacher will ask children to describe the final question being posed.
Teacher asked children to solve problem in ways that makes sense to them	In an effort to help students solve the story problem in any way that makes sense to all children, the teacher explicitly states a similar statement "you can solve the problem in any way that makes sense to you" to the children. This is done to help each child solve using their own authentic strategies instead of using previously learned strategies from their math classroom.
Categories of Teacher practices during problem solving	

Teacher elicits children's thinking about their current strategies	*Teacher invites individual or a pair of children to explain the strategies they used*	Teacher uses general questions like "Can you explain how you solved it?" or "Would you tell me what you did to solve the problem?" to help in understanding how the children solved the story problem.
	Teacher provides prompts or questions to help children explain specific details about their problem solving strategies	Teacher attends to the details of the children strategies used and asks specific questions about certain steps in a child or group of children's strategies. The teacher usually asks children do describe a particular part of their strategy (e.g." I saw that you added five each time, why did you do that? How did that help you?")
Teacher ask for clarification of children's thinking about their current strategies and understanding of the problem to help them make sense of their thinking		Teacher notices child or group of children are struggling in solving the story problem. Teacher engages in conversation with a child or group of children about describing what the story problem is about. The teacher usually asks questions that help clarify children's current understanding of the problem and provide appropriate prompts (e.g. "What is the problem about?" followed by "I noticed you added 4 to each of the boxes, does that help you make sense of what is the problem asking you to do?") to help children link the story problem and the details of their current strategy.
Teacher assigns competence to children's ideas	*Teacher revoicing children's descriptions of their strategies to assign validity to their strategies.*	Teacher rephrases what a child or group of children said when explaining their strategies used for the story problem in order to help them understand their thinking process and assign competence to their strategies as valid strategies.

	Teacher revoicing a child's descriptions of his/her strategy to a peer to assign them as an expert among a peer or group of peers	The teacher usually asks a pair or group of three to engage in a conversation about the strategies they used. This conversation is usually followed with the teacher rephrasing what a child said when explaining his/her strategy to a peer in order to assign this child the role of the expert. Thus, the teacher positions the child as a problem solver and a contributor of math ideas.
	Teacher prompts a child to justify his/her agreement with a peer's strategy to assign them as justifiers of mathematical ideas (Turner et al. 2013)	When pairs or groups of children are explaining each other's strategies, the teacher asks one of the child in the group to justify how their peer's strategy works. The teacher usually asks a question like "Could you explain to us how do you know that Maria's strategy works?" Thus, the teacher positions the child as the justifier of mathematical ideas.
	Teacher invites a child to evaluate his/her disagreement with a peer's strategy to assign him/her as an evaluator of mathematical ideas.(Turner et al. 2013)	When pairs or groups of children are discussing each other's strategies, the teacher asks one of the child in the group to evaluate his/her disagreement with a peer's strategy. The teacher usually ask a question like "I think Maria disagrees with Jose's math descriptions or ideas, Maria could you tell me why you are shaking your head no?". Thus, the teacher positions the child as the evaluator of mathematical ideas.
Teacher extends	*Teacher solicits a different*	The teacher encourages individual or group of children to solve the story problem using a different strategy than the prior one or to

children's thinking	*strategy than the prior one*	compare a different strategy with a peer from their own.
	Teacher asks children to use a number sentence to represent their strategies	The teacher prompts children to create a number sentence that describes their own strategies, solutions or math ideas using a mathematical equation or expression.
	Teacher asks follow up problems that have more challenging numbers	Teacher introduced a new problem with the same story context to an individual or group of children. The story problem is usually the same problem but with different more challenging numbers. An example would be changing this problem form " Amelia has 13 jellybeans. Her brother gives her 8 more jellybeans. How many jellybeans does Amelia have now?" to "Amelia has 17 jellybeans. Her brother gives her 9 more jellybeans. How many jellybeans does Amelia have now? "
Categories of Teacher Practices during group discussion		
Teacher elicits children's thinking about their current strategies	*Teacher invites individual or a pair of children to explain the strategies they used*	Teacher uses general questions like "Can you explain how you solved it?" or "Would you tell me what you did to solve the problem?" to help in understanding how the children solved the story problem.
	Teacher provides prompts or questions to help children explain specific details about their problem solving strategies	Teacher attends to the details of the children strategies used and asks specific questions about certain steps in a child or group fo children's strategies. The teacher usually asks children to describe a particular part of their strategy (e.g." I saw that you added five each time, why did you do that? How did that help you?")

Teacher assigns competence to children's ideas	*Teacher revoicing children's descriptions of their strategies to assign validity to their strategies.*	Teacher rephrases what a child or group of children said when explaining their strategies used for the story problem in order to help him/her or them understand their thinking process and assign competence to their strategies as valid strategies among the group of children.
	Teacher revoicing a child's descriptions of his/her strategy to the group to assign them as an expert among the group.	The teacher usually asks an individual or a pair of children to describe their strategies or math ideas to all children in the group. This discussion is usually followed with the teacher rephrasing what a child/pair of children said when explaining his/her or their strategy to the group in order to assign this child or pair the role of the expert(s). Thus, the teacher positions the child or pair as (a) problem solver(s) and as contributor(s) of math ideas.
	Teacher prompts a child to justify his/her agreement with a peer's strategy within the group discussion to assign them as justifiers of mathematical ideas (Turner et al. 2013)	When pairs or groups of children are explaining each other's strategies within the group discussion, the teacher asks one of the children in the group to justify how their peer's strategy works. The teacher usually asks a question like "Could you explain to us how do you know that Maria's strategy works?" Thus, the teacher positions the child as the justifier of mathematical ideas.
	Teacher invites a child to evaluate his/her disagreement with a peer's	When pairs or groups of children are articulating each other's strategies within the group discussion, the teacher asks one of the child in the group to evaluate his/her disagreement with a peer's strategy. The teacher usually ask a question like "I think Maria disagrees with Jose's math

	strategy to assign him/her as an evaluator of mathematical ideas.(Turner et al. 2013)	descriptions or ideas, Maria could you tell me why you are shaking your head no?". Thus, the teacher positions the child as the evaluator of mathematical ideas.
Teacher extends children's thinking	*Teacher solicits a different strategy than the prior one*	The teacher encourages individual or group of children to compare a different strategy with a peer from their own.
	Teacher asks children to use a number sentence to represent their strategies	The teacher prompts children to create a number sentence that describes their own strategies, solutions or math ideas using a mathematical equation or expression.

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